

# Competence building with the use of nurse re-rostering

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**Abstract.** The global nursing shortage makes efficient use of these resources vital. Good nurse rosters assist but are often static and span over a long period while the daily personnel situation is more dynamic: for example, nurses get sick or take short notice days off. Commonly, these absences are handled by hiring extra nurses when needed. However, earlier analysis has shown that nurse rotation in combination with hiring is a much more efficient solution when there exist a tool that efficiently re-rosters already planned nurses. Moreover, re-rostering gets easier if the hospital possesses the best mix of experience level and special skills. In other words, a more suitable competence profile makes re-rostering more beneficial. Nurse rotation (regularly working in another department) builds up competence, which allows for a more robust competence profile—departments become better suited to handle future personnel absences. We present a prototype that optimizes the competence profile using a stochastic model: it finds the optimal competence profile under the assumption that nurse rotation is allowed and the hospital can buy in competence. Each profile is evaluated by different absence scenarios. Our preliminary experiments show how a more robust competence profile is able to result in a 40 % cost reduction for the hospital while retaining at minimum the quality of care.

**Keywords:** nurse rostering; stochastic programming; competence building

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## Introduction

With the hospital sectors high cost and nursing shortage, good utilization of existing nurses becomes imperative. Building efficient nurse rosters is important and should

incorporate skills, competence, laws, and union regulations. Normally these span over several months and are static while the daily personnel situation is much more dynamic—employees get sick, take short notice days off, or the demand increases due to emergencies. In Norway, these absences are normally filled with temporary workers, who are very expensive and inefficient according to our reference hospital A-HUS (2011). The temporary workers often lack the desired competence, knowledge about hospital layout, routines, and procedures. So, regular staff uses time to instruct them—time that could be better spent on patient care instead. Our reference hospital has introduced the concept of intelligent nurse rotation (instead of solely hiring) supported by a re-rostering support tool. An earlier analysis has indicated that this is indeed very beneficial for A-HUS. For a 10 % absence ratio, the hospital could reduce total hiring costs by 50 %, which is equivalent to 10 % of its total wage costs. However, A-HUS wants to improve this idea by determining the optimal competence profile of its nurses, such that the hospital becomes better equipped to handle nurse absences through nurse rotation. In this paper, we develop a prototype of a strategic competence building tool in order to assess potential benefits of an optimal competence profile.

The core contributions of this paper are:

1. Optimal competence building is an important but mostly unattended research domain.
2. The model creation helps understanding the problem the hospital is facing.
3. The prototype is generic and could be applied to other sectors as well.
4. The prototype is developed in close cooperation with a Norwegian hospital, and tested on realistic instances based on discussions with the hospital.

The paper is organized as follows: Section 2 provides the background information and related research. The problem is introduced in Section 3. In Section 4, we describe the experiment setup and discuss the results. We conclude and present the planned future work in Section 5.

## Background

Nurse rostering is the process of creating a work schedule for hospital nurses by matching employees to shifts over a given planning horizon, while considering skills, competence, fairness, laws and regulations, and sometimes preference. The output is a roster of the working hours for the nurses that also provides an overview of staff utilization and associated costs. Nurse rostering problems are combinatorial optimization problems that in most cases are NP-hard (Karp, 1972). For a more comprehensive overview of the nurse rostering literature, see Burke et al. (2004).

However, in this case, we start from an initial roster. Thus, the focus lies on short-term re-rostering in respond to dynamic events. Like many other rescheduling problems, re-rostering problem has received little attention (Pato and Moz, 2007). Warner (1975) highlighted the need for re-rostering, but never developed any algorithms for it. Bard and Purnomo (2005) published the first article on short-term nurse scheduling due to daily fluctuations in supply and demand, using integer

programming. Later, genetic algorithms (Pato and Moz, 2007) and ant colony optimization (Gutjahr and Rauner, 2007) has been applied. The literature shows that different solution methods have been applied and some models incorporate preference constraints, pool of nurses, and competence. Still, none of them use the strategic competence building opportunity that re-rostering provides.

## Problem description

The model must take into account the uncertainty originating from day-to-day absences which are unknown when the decision of which competences to be built for each nurse, is made. Stochastic programming is a framework for modelling problems with this type of uncertainty. In stochastic programming, uncertainty is often modelled through the use of scenarios, representing a fair part of possible outcomes, and possible recourse actions. These actions compensate for any bad effects occurring when uncertain parameters (in this case nurse absences), become certain. These models are called recourse models and we apply this framework to our problem. Therefore, we generate a number of absence scenarios to assess the expected daily costs for a specific competence profile by hiring external nurses or moving nurses to other wards in the most efficient way. The expected daily cost for each competence profile is the sum of the probability of a scenario occurring multiplied with the daily costs of the respective scenario. Hence, the re-rostering model has become an integral part of our model. The objective and constraints of our model are described hereafter:

The hospital needs to balance the extra long-term costs corresponding to a higher competence profile with the reduction in operational costs (costs of hiring external competence). Therefore, the objective function (1) consists of two main parts. The first part is related to the more strategic decisions of building up a certain nurse competence profile for a longer time period. It is the sum of extra salary for the nurses who gained competence by experience, costs of bought competence, as well as extra salary and costs of bought skill. The second part (evaluation) represents the daily expected costs, which consists of three terms: costs of hiring external nurses, extra cost of hiring nurses holding special skill, and the administration costs for nurses rotating to other wards.

$N$	set of all nurses listed in the given shift
$N_S^P$	set of listed nurses who are present in the evaluation shift in scenario $s$ , $N_S^P \subseteq N$
$W$	set of wards
$L$	set of competence levels
$\varepsilon$	set of special skill
$\varepsilon_w$	set of special skill that are demanded in ward $w$
$S$	set of scenarios in the evaluation phase
$C_n^G$	the additional cost per time period for increased competence level for nurse $n$

$C_n^U$	the additional cost per time period for buying nurse $n$ extra competence
$C_n^M$	the additional cost per time period for buying nurse $n$ special skill
$C_{nw}^{ADMIN}$	administration cost of having nurse $n$ rotated to ward $w$
$C_{wl}^E$	cost for hiring a nurse externally with competence level $l$ at ward $w$
$C_{we}^{EE}$	additional cost for hiring a nurse externally with special skill $e$ at ward $w$
$A_{ns}$	1 if nurse $n$ is absent in scenario $s$ , 0 otherwise
$Q_{nw}$	the initial competence level that nurse $n$ has in ward $w$
$G_{nw}$	maximum competence level nurse $n$ can gain with experience at ward $w$
$G_n^{MAX}$	the total maximum of gained competence levels for nurse $n$
$U_{nw}$	maximum competence levels the hospital can buy for nurse $n$ in ward $w$
$U_n^{MAX}$	the total maximum of bought competence levels for nurse $n$
$O_n^{MAX}$	the total maximum of bought special skill for nurse $n$
$D_{wl}^{LA}$	aggregated demand for nurses at ward $w$ with level $l$
$D_{we}^E$	demand for nurses with special skill $e$ in ward $w$
$E_{ne}^E$	1 if nurse $n$ has the special skill $e$ , 0 otherwise
$P_s$	probability of scenario $s$
$g_{nw}$	gained competence level(s) for nurse $n$ in ward $w$
$u_{nw}$	bought competence level(s) for nurse $n$ in ward $w$
$q_{nw}$	new competence level for nurse $n$ in ward $w$
$o_{ne}$	1 if bought special skill $e$ to nurse $n$ , 0 otherwise
$m_{ne}$	new value of special skill: 1 if nurse $n$ has skill $e$ , 0 otherwise
$k_{nwes}$	1 if nurse $n$ works at ward $w$ in scenario $s$ and has the special skill $e$ , 0 otherwise
$x_{nwls}$	1 if nurse $n$ ends up working in ward $w$ with level $l$ at the given time period and scenario $s$ , 0 otherwise
$y_{wls}$	number of nurses with competence level $l$ which are hired externally to ward $w$ at the given time period and scenario $s$
$z_{wes}$	number of nurses with special skill $e$ , which are hired externally to ward $w$ at the given time period and scenario $s$

**Complete model.** The objective together with the definitions stated above gives the following model:

$$\min \sum_{n \in N} \sum_{w \in W} C_n^G g_{nw} + \sum_{n \in N} \sum_{w \in W} C_n^U u_{nw} + \sum_{n \in N} \sum_{e \in \varepsilon} C_n^M o_{ne} + \sum_{s \in S} \left( P_s \left( \sum_{w \in W} \sum_{l \in L} C_{wl}^E y_{wls} + \sum_{w \in W} \sum_{e \in \varepsilon} C_{we}^{EE} z_{wes} + \sum_{l \in L} C_{nw}^{ADMIN} x_{nwls} \right) \right) \quad (1)$$

Subject to:

$$Q_{nw} + g_{nw} + u_{nw} = q_{nw}, \quad n \in N, w \in W \quad (2)$$

$$E_{ne}^E + o_{ne} = m_{ne}, \quad n \in N, e \in \varepsilon \quad (3)$$

$$g_{nw} \leq G_{nw}, \quad n \in N, w \in W \quad (4)$$

$$u_{nw} \leq U_{nw}, \quad n \in N, w \in W \quad (5)$$

$$\sum_{w \in W} g_{nw} \leq G_n^{MAX}, \quad n \in N \quad (6)$$

$$\sum_{w \in W} u_{nw} \leq U_n^{MAX}, \quad n \in N \quad (7)$$

$$\sum_{e \in \varepsilon} o_{ne} \leq O_n^{MAX}, \quad n \in N \quad (8)$$

$$q_{nw} \geq 0, \text{integer}, \quad n \in N, w \in W \quad (9)$$

$$g_{nw} \geq 0, \text{integer}, \quad n \in N, w \in W \quad (10)$$

$$u_{nw} \geq 0, \text{integer}, \quad n \in N, w \in W \quad (11)$$

$$m_{nw} \in \{0, 1\}, \quad n \in N, w \in W \quad (12)$$

$$o_{nw} \in \{0, 1\}, \quad n \in N, w \in W \quad (13)$$

$$\sum_{w \in W} \sum_{l \in L} x_{nwls} = 1 - A_{ns}, \quad n \in N_s^P, s \in S \quad (14)$$

$$\sum_{l \in L} l x_{nwls} \leq q_{nw} \quad n \in N_s^P, \quad w \in W, s \in S \quad (15)$$

$$-\sum_{l \in L} x_{nwls} + k_{nwes} \leq 0, \quad n \in N_s^P, w \in W, e \in \varepsilon, s \in S \quad (16a)$$

$$-m_{ne} + k_{nwes} \leq 0 \quad n \in N_s^P, \quad w \in W, e \in \varepsilon, s \in S \quad (16b)$$

$$\sum_{n \in N_s^P} \sum_{l' \in L: l' \geq l} x_{nwls} + \sum_{l' \in L: l' \geq l} y_{wls} \geq D_{wl}^{LA}, \quad w \in W, e \in \varepsilon, s \in S \quad (17)$$

$$\sum_{n \in N_s^P} k_{nwes} + z_{wes} \geq D_{we}^E, \quad w \in W, e \in \varepsilon_w, s \in S \quad (18)$$

$$z_{wes} \leq \sum_{l \in L} y_{wls}, \quad w \in W, e \in \varepsilon_w, s \in S \quad (19)$$

$$x_{nwls} \in \{0, 1\}, \quad n \in N_s^P, w \in W, l \in L, s \in S \quad (20)$$

$$y_{wls} \geq 0, \text{integer}, \quad w \in W, l \in L, s \in S \quad (21)$$

$$z_{wes} \geq 0, \text{integer}, \quad w \in W, e \in \varepsilon_w, s \in S \quad (22)$$

$$k_{nwes} \in \{0, 1\}, \quad n \in N_s^P, w \in W, e \in \varepsilon, s \in S \quad (23)$$

**Building constraints.** Constraints (2) and (3) keep track of the building of competence and special skill respectively. Constraints (4) and (5) state that the gained, bought and lost competence levels do not exceed the maximum level for each ward and nurse, while constraints (6) to (8) ensure that each nurse do not exceed the number of possible gained and bought competence levels and bought special skill. Actually, these constraints indirectly define the time horizon in which the competence profile is to be built. The higher the indicated maximum levels, the longer the time horizon can be. Finally, constraints (9) to (13) define the building phase variables as integer, binary and non-negative.

**Link and evaluation constraints.** Constraint (14) ensures that each nurse can only work at one ward in the given shift, if he or she is available for a specific scenario. Constraint (15) restricts the nurse to work in a ward with a higher competence level than he or she holds in the new competence profile. Hence, these constraints link the building phase and the evaluation phase. Constraints (16a,b) connect the special skills of a nurse,  $m_{ne}$ , and where he or she works,  $x_{nwls}$ , with the variable  $k_{nwes}$  specifying that the special skill of the nurse can be used in the given ward.

Constraints (17) state that the minimum demand is fulfilled in all the combinations of ward, the hierarchic competence levels and scenario, while constraints (18) ensure the minimum demand of special skill in each ward and scenario. Constraints (19) specify that each hired nurse can only have one special skill. Finally, the constraints (20) to (23) define the variables as binary, integer and non-negative.

## Empirical studies

**Setup experiment:** Since the main goal of the paper is to measure the potential (financial) impact of nurse rotation with the opportunity of competence building for the hospital, and not finding the most efficient algorithm, not much effort has been spent on building a specialized procedure to solve this problem. A general-purpose solver, XPRESS-MP (v. 7.2) from FICO, is used to solve several instances to optimality with a maximum time limit of 36 000s.

Data for the experiment was extracted from a department at A-HUS. Originally, the instances were of larger size, but it was needed to reduce the problem size due to computational limitations, until it comprised only 4 instead of 8 wards in which approximately 60 patients are treated and 28 nurses are working in three shifts (day, evening, night). Currently, only one shift (day shift) is considered. Despite these simplifications, the current experimental instance is still believed to be large and complex enough in order to gain insight in this new type of problem. The instances can be made available by the authors upon request.

An earlier analysis has indicated that on average between 8 and 14 per cent of the nurses are absent each day at the hospital. Therefore, we have generated 30 scenarios with an absence rate of 10 per cent lying within the aforementioned interval. As mentioned, these scenarios are used to evaluate how well a specific competence profile can cope with the day-to-day absences. The nurses themselves can have 4 levels of experience and/or 10 kinds of special skills, half of which are ward-dependent while the other half is useful across all wards. The different cost parameters are set in accordance with the hospital salary levels and are scaled down to an extra salary cost for a shift so that it can be compared with the expected costs of hiring and rotation nurses in one shift. The relevant competence building period is set to 3 to 6 months. Given this period, nurses are only able to increase one experience level through on-the-job experience or following courses (bought experience) at a time and to obtain one special skill. The minimum aggregated demand of needed special skills and experience level for each ward has been collected backward by investigating last year's manual rosters.

Firstly, the basic analysis demonstrates how much can be gained by adapting the overall competence profile and how the solutions are structured. Secondly, supplementary analyses were conducted to investigate further the usefulness of competence building in the hospital.

**Basic analysis:** Recently, Lilleby (2012) has shown for the same hospital for a 10%-absence ratio that nurse rotation without competence building can already reduce hiring costs by 50 %, which is equivalent to 10% of the total costs (including wage costs).

In addition to this observation, the experiments in this paper reveal that an optimal competence profile can reduce the total costs by an extra 40% including the increased wage costs of the higher competence profile. The results show that 8 nurses gained competence by experience, while there was no bought competence. The reason for no competence was bought in this specific case was the higher competence cost and that the existing competence structure allowed enough competence gained by

experience. Also, the nurses acquired 6 extra special skills. Details of the competence building proved that nurses mainly increase their competence in their home wards, including both gained competence and bought special skill. Also, we observed more rotations of nurses between the different wards when the optimal competence profile was used compared to the initial profile. Overall, the higher rotation rate led to lower costs of external hires and thus decreased the costs of absence and total costs.

**Supplementary analysis:** One of the advantages of the tool is that it pinpoints in which ward and how much competence needs to be increased. This is very valuable as a human resource policy tool, for example, as a starting point to discuss career planning. However, further analysis of the best 20 solutions shows that there exists no difference in how much competence is build and bought in. The solutions differ mainly with respect to how the absences are handled during the evaluations of the scenarios. Different nurses are rotated, and the external hires are in other wards and levels, but the numbers of rotated nurses and hired nurses are the same as in the optimal solutions. One of the reasons of having a large number of similar, optimal solutions could be because the salary increase as a consequence of gained experience is the same for each level and undependable on the starting level. More distinct wage increases depending on the gained experience level could trigger less similar optimal solutions, but this should be investigated further. Nevertheless, this means that probably there is room for incorporating nurse preferences towards rotation. Some nurses are more eager to rotate than others.

Furthermore, some tests were conducted with an increased maximum limit of gaining competence up to 6 levels, thereby implying a longer competence build up period, to see whether higher competence gains can decrease total wage costs even further. These tests demonstrated that the latter assumption is not the case, and that the hospital can reach an optimal competence profile by even a relatively small investment (+1 experience level, +1 extra special skill) for a 10% absence level.

Finally, it is worth investigating how the reduction in total wage costs behaves when absence ratio is lower or higher than the default 10%-absence ratio. The results of this extended analysis indicate that the relative reduction in total wage costs decreases when increasing the absence ratio. The absolute reduction stays similar to what is expected around 10 % absence ratio. At the highest 20%-absence ratio it shows a cost reduction of 10 %, which can hardly be neglected by hospital management. Moreover, probably more can be gained from higher increases in experience level and skill level than what has been experimented with currently, but this has to be tested more thoroughly in the future. The resulting cost reduction is much lower under 5%-absence ratio.

## **Conclusion and future work**

In this paper, we presented a novel approach for assisting operational as well as strategic staff planning activities through optimizing nurse competence profiles. We use a stochastic model that builds optimal competence profiles if nurse rotation and/or purchase competence is possible. Each profile is evaluated by applying

different absence scenarios to determine how well they cope with different absences each day. We have tested our approach using instance data based on input from our reference hospital, and presented the results in this paper: our experiment indicate that optimized competence profiles can improve nurse utilization, lowering the cost, assist in personnel career planning and build robust competence structures across departments. For future work, we wish to explore specialized algorithms and develop the prototype to handle more nurses and more scenarios, nurse preference, and nurse pooling. Because, the basic idea behind it is very generic, we plan to apply it in the domain of air traffic management to roster air, ground and runway controllers.

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