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A simplified model for scheduling services on auxiliary bus lines

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Abstract. In this paper, a mathematical programming based model is described to assist with the schedule of services for a set of auxiliary bus lines that operate alleviating a disruption of the regular transportation system during a given time period. In contrast to other models, considered static, service schedules are set taking into account demand fluctuations that may happen in that time period. Passenger flows are represented with a multi-commodity structure and disseminate through paths on a diachronic capacitated network which lead them to their destination in the shortest time taking into account the available capacities of the bus units. The model permits to accommodate the schedules of the auxiliary bus lines in order to enhance transfers and to minimize total travel time. The model assumes that operational times at bus stops are constant and that buses do not queue at stops. The solution method can be considered an heuristic that combines searches along subgradient's projected directions with a pattern search based method for constrained optimization.

Keywords: public transportation; service scheduling; disruption management; auxiliary bus lines

Introduction

Disruptions in mass transportation systems are the cause of delays, annoyances and discomforts in the mobility of cities and metropolitan areas. One means of reducing the effects of disruptions is to bridge the stations at disrupted sections of the regular service (either rapid transit or metro) with auxiliary transportation services that must operate under high-demand conditions. Another problem that these auxiliary services must face are the peak flows originated at disrupted stations on the arrival of trains or by unexpected fluctuations that may be only predicted with little anticipation. These fluctuations superimpose to the expected daily variability of the demand making that a planning of a time table of auxiliary services based on expected values for the demands can be unappropriate resulting in delays that can be still minimized. Thus, the synchronization of services and the adaptation of transfer times between auxiliary lines and regular services is of utmost relevance.

Establishing auxiliary bus services has been a common practice, but only recently, in Kepaptsoglou and Karlaftis (2008), has this problem been focussed directly. Also, in Codina *et al.* (2013) a model for planning auxiliary bus lines is developed which considers explicitly the degree of congestion of the auxiliary system and its possible bottlenecks. The models in these works can be considered static since just average demand values are taken into account and frequencies, or total number of services during the period, are determined instead of service schedules oriented to the possible synchronization of arrivals and transfers. However, although dynamic transit assignment scheduled oriented models have been developed, such as in Papola *et al.* (2009) or in Haupt *et al.* (2009), they are difficult to integrate into a frequency/schedule setting framework. Also, models for time table setting in railway systems that minimize a weighted combination of time transfers have been extensively studied, for instance in Schöbel (2006), as well as algorithms for solving them (see, for instance Liebchen *et al.* (2008)). Scheduled based transit assignment has also extensively studied taking into account stochastic models for route choice in Tong and Wong (1999) and FIFO flow observance Nguyen *et al.* (2001).

In this paper a simplified scheduled based transit assignment model is developed which can be used efficiently for the rescheduling of services in a dynamic context for a public transportation network of moderate size, such as the auxiliary systems used for alleviating disruptions. It is based on a capacitated multicommodity flow problem on a diachronic network covering a fixed time period for which a dynamic origin destination matrix is assumed to be available, capturing instantaneous bulk flow arrivals at stations of the disrupted transportation system. Time schedules are determined using an heuristic algorithm that uses search directions that are projected subgradients of an objective function minimizing the total passengers travel time of the system. When search along these directions fail then, an adaptation of the pattern search based method by Lewis and Torczon (2000) is used to determine a valid movement or to stop the algorithm.

Model's description

A prior step to the application of the model described in this paper would be to set up the structure of the auxiliary transport system, i.e., its stops and line layouts as well as an approximate number of services that need to operate on that lines. For this purpose, the layout of the disrupted system must be taken into account and its relationship with the urban network topology. Thus, it must be studied whether the stations of the disrupted system can be served by a single stop or by separate stops accordingly to the possibilities that offer the configuration of the urban network on which the auxiliary system must operate. Next, it must be determined which lines can be allocated on the auxiliary system accordingly to the street and bus stop capacities by means of a network design model using mean values for demands and operational times, such as, for instance the one developed by Kepaptsoglou and Karlaftis (2008) or, if the auxiliary system must operate under high congestion levels, then the one by Codina *et al.* (2013).

The model assumes that the layout of the auxiliary bus lines system is fixed and known, i.e., for each line the sequence of stops to be performed is known as well as the total number of services on that line. Then, what the model seeks to determine is the time schedule of each of these services, or equivalently, the time lags between consecutive services with the purpose of minimizing trip travel times of users of the auxiliary transport system and enhance the possible internal transfers of the auxiliary system as well as between the auxiliary system and the portion of the regular system still operative. In order to obtain a better computational performance two basic assumptions have been made: *a*) there is no queueing at bus stops of the auxiliary transport system and *b*) boarding/alighting times are fixed to reasonable values independently of boarding/alighting flows.

Structure of the diachronic network

The core of the model is a diachronic network where links are associated to possible passenger movements and movements of units in a service. Throughout the text, double indexed elements such as links in a graph will be denoted either by a single letter a or by the labels of the tail and head nodes, (i, j) at convenience. The same will apply for origin destination (o-d) pairs, that will be usually designated by ω or explicitly by (p, q). For a better description of the diachronic network, let $\mathbf{G} = (\mathbf{N}, \mathbf{A})$ be a planar directed graph representing the spatial distribution of the auxiliary transportation system. As a convenient notation, all elements associated to the planar graph will be written in bold. In order to define the trip demands, it will be assumed that a subset of the nodes $\mathbf{C} \subseteq \mathbf{N}$ are centroids on which a set of o-d pairs $\mathbf{W} \subseteq (\mathbf{C} \times \mathbf{C}) \backslash \{ (\mathbf{p}, \mathbf{p}) \mid \mathbf{p} \in \mathbf{C} \}$ is defined. The subset of centroids that are origins and destinations will be denoted by $\mathbf{O}$ and $\mathbf{D}$ respectively. Let $\mathbf{S} \subseteq \mathbf{N}$ the set of stops of the auxiliary transport system. In general, $\mathbf{S} \cap \mathbf{C} \neq \emptyset$. Let also $\tilde{L}$ denote the set of transport lines operating on G; then $\tilde{L} = L \cup L^0$, where L and L^0 will stand for the set of auxiliary lines and the set of regular lines included in the scenario, respectively. The sequence of stops in line $\ell \in \tilde{L}$ will be denoted by $\mathbf{S}(\ell)$, the segments making up line ℓ by $\mathbf{A}(\ell)$ and the set of lines halting at a stop $\mathbf{i} \in \mathbf{S}$ will be denoted by $\tilde{L}(\mathbf{i})$. Let n_ℓ the number of services initially expected for line $\ell \in L$ and let $\delta_{\ell,s}$ be the time lag between service $s - 1$ and s at the initial stop of line $\ell \in L$. Time lags for lines $\ell \in L^0$ are assumed constant and left at their initial settings. The vector of time lags for line $\ell \in L$ will be denoted by δ_ℓ, whereas δ will denote the vector for time lags of all lines of the auxiliary transport system.

Let H be the time horizon covered by the model (typically 2 to 3 hours). The diachronic network will be associated to a time granularity given by a small time subinterval Δ, typically of $<= 1$ minute. The graph for the diachronic network will be denoted by $G = (N, A)$. Nodes $i \in N$ are assigned a time label t_i which may be constant or which may be determined by the decision variables δ. The set of links A is divided into $A = A_x \cup A_e \cup A_T \cup A_a \cup A_y \cup A_P \cup A_g$. Links in each of these subsets are assigned specific travel cost functions and play different roles as shown in figure 1 below. The role of links in subset A_g will be explained further in subsection 2.3. The forward and backward stars of links emerging/incoming from/at node $i \in N$ will be denoted as $E(i)$ and $I(i)$ respectively.

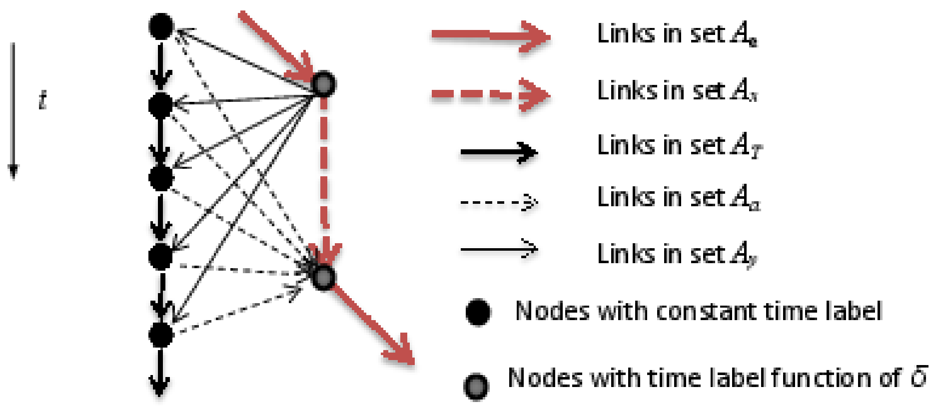

Fig. 1. Graphical description of the topology of the diachronic network $G = (N, A)$.

Passenger flows on links $a \in A_x$ are dwelling at a stop during boarding/alighting operations; flows on links $a \in A_e$ are moving through a line segment from station to station; flows on links $a \in A_a$ are boarding from a stop whereas flows on links $a \in A_y$ are descending to a stop. Transfers and other external movements are captured by links $a \in A_P$. Passenger flows waiting at stops are captured by links $a \in A_T$. By $A_x(\mathbf{i}), A_y(\mathbf{i}), A_a(\mathbf{i}), A_T(\mathbf{i})$ it will be denoted the corresponding subsets of links associated to node $\mathbf{i} \in \mathbf{S}$. Likewise, $A_x(\ell), A_y(\ell), A_e(\ell), A_a(\ell)$ will denote subsets associated to line $\ell \in \tilde{L}$, $A_x(\ell, s), \ldots$ the subsets associated to service s for line $\ell \in \tilde{L}$ and $A_x(\mathbf{i}, \ell), \ldots$ will denote links associated to node i and line ℓ. A node $i \in N$ which is head or tail of a link in A_x, A_y, A_a or A_T, has associated a stop in the planar graph that will be denoted by $\mathbf{i}(i)$.

For each node $\mathbf{i} \in \mathbf{N}$ in the planar graph, the diachronic graph G contains a linear sequence of links $a \in A_T(i)$, each of them with a travel cost Δ. Their head and tail nodes j_a and i_a, respectively, are assigned a time label that is a multiple of Δ. For a node $\mathbf{i} \in \mathbf{S}$, in the planar graph, the set of nodes that are head and tail for links in $A_T(i)$ will be denoted by $N_T(\mathbf{i}) = \{j_a, i_a | (i_a, j_a) \in A_T(\mathbf{i})\}$. These nodes appear in bold in figure 1; passenger flows on these links correspond to waiting at node $\mathbf{i} \in \mathbf{N}$. Each service in a line $\ell \in \tilde{L}$, i.e., the journey of a bus or transport unit along the line, is represented by a sequence of links $a_1, a_2, \ldots, a_\nu \in A$, so that $a_{j-1} \in A_e, a_j \in A_x, a_{j+1} \in A_e \ldots$ and $a_1, a_\nu \in A_e$. Head and tail nodes for the links in this sequence appear in grey in figure 1 and their time label is a function of the decision variables δ if the line $\ell \in L$, or is constant if $\ell \in L^0$. If $a \in A_e$, its head node, $j_a \in N$, will be associated to a stop $\mathbf{i} \in \mathbf{S}$ and its tail node, $i_a \in N$, to another stop $\mathbf{i}' \in \mathbf{S}$. Node j_a is assumed connected to any node in $N_T(\mathbf{i})$ by means of links $a' \in A_a$. Also, node i_a is assumed connected to any node in $N_T(\mathbf{i})$ by means of links $a'' \in A_y$.

Link cost functions

Link cost functions for the links in the diachronic graph G will depend on link travel times at line segments of the planar graph $\mathbf{G}$ and operational times at stops. Let $\tau_{\mathbf{i}}^{\ell}$ the time to reach station $\mathbf{i} \in \mathbf{S}$ from the beginning of line ℓ calculated on the planar graph $\mathbf{G}$ taking into account operational times at previous stops of the line and travel times on line segments. Then, for any link $(i,k) \in A_x(\ell)$, $\mathbf{i}(i) = \mathbf{i}(k)$, an initial arrival time label $t_{i,\ell}^0 = \tau_{\mathbf{i}(i)}^{\ell}$ will be associated to node i and an initial exit time label $t_{k,\ell}^0 = \tau_{\mathbf{i}(k)}^{\ell} + x_{\mathbf{i}(k),\ell}$ will be associated to node k.

Link cost functions for links $a \in A$ are the following ones:

- if $a \in A_x$ then $c_a = x_{\mathbf{i},\ell}$ is the operational time of transport units in line ℓ at stop $\mathbf{i} \in \mathbf{N}$. It may be assumed fixed to a constant value or it may be a function of the boarding and alighting flows.
- if $a \in A_e$ then c_a is the in-vehicle time or travel time for the line segments.
- if $a \in A_T$ then $c_a = ctant = \Delta$ is the time subinterval marked by the time granularity of the model.
- if $a \in A_a$ or $a \in A_y$ then the cost functions c_a, $a = (i,j)$, for these links are assumed to be flow independent and are parametrized by the difference of their time labels $\theta_{ij} = t_j - t_i$ of their head and tail nodes, and are given by:

$$c_{ij}(\theta_{ij}) = max\left\{H(1 - \frac{\theta_{ij}}{z_0}),\ \theta_{ij},\ H(1 - (\frac{x_{\mathrm{i}(i),\ell} - \theta_{ij}}{z_0}))\right\} + \frac{\varepsilon}{2} \quad (1)$$

where ε is a predefined small number and z0 can be adopted as a small fraction of the operational time $x_{\mathbf{i},\ell}$ at stops (i.e, < 5%). By adding $\varepsilon/2$ the following convenient condition is achieved:

$$c_{ij}(\theta_{ij}) + c_{jk}(\theta_{jk}) > x_{\mathrm{i}(i),\ell}\ ,\quad (i,k) \in A_x(\mathbf{i}(i),\ell)\ \&\ j \in N_T(\mathbf{i}(i)) \quad (2)$$

The time instant labels for nodes i,j in previous expression (1) will be evaluated by means of the initial arrival and exit times by means of:

$$\mathbf{i} \in \mathbf{S}, \qquad \ell \in L(i), 1 \le s \le n_\ell:$$
$$(i,j) \in A_y(\mathbf{i},\ell,s) \Rightarrow \begin{cases} t_i = t_{i,\ell}^0 + \sum_{\sigma=1}^{s} \delta_{\ell,\sigma} \\ t_j = \text{fixed} \end{cases},$$
$$(j,k) \in A_a(\mathbf{i},\ell,s) \Rightarrow \begin{cases} t_k = t_{k,\ell}^0 + \sum_{\sigma=1}^{s} \delta_{\ell,\sigma} \\ t_j = \text{fixed} \end{cases} \quad (3)$$

being $A_y(\mathbf{i},\ell,s) = A_y(\ell,s) \cap A_y(\mathbf{i},\ell)$ and $A_a(\mathbf{i},\ell,s) = A_a(\ell,s) \cap A_a(\mathbf{i},\ell)$. Assume that for a link $a = (i,k) \in A_x(\mathbf{i})$, the time labels t_i and t_k are fixed with $t_k - t_i = x_{\mathbf{i}(i),\ell}$; then, only the links $(i,j) \in A_y(\mathbf{i})$ that need to be considered are those verifying

$t_i \leq t_j \leq t_i + x_{\mathbf{i}(i),\ell}$ and the links $(j,k) \in A_a(\mathbf{i})$ that need to be considered are those verifying $t_k - x_{\mathbf{i}(k),\ell} \leq t_j \leq t_k$, because all other links result in a cost $c_a \geq H$. . Let $A_y(i)[t_i, x_{i,\ell}]$ and $A_a(i)[t_k, x_{i,\ell}]$ denote the set of links verifying these conditions. It is not difficult to show that if $\Delta < min_{i,\ell}\{x_{i,\ell}\}$ then, $A_y(i)[t_i, x_{i,\ell}]$ and $A_a(i)[t_k, x_{i,\ell}] \neq \emptyset$.

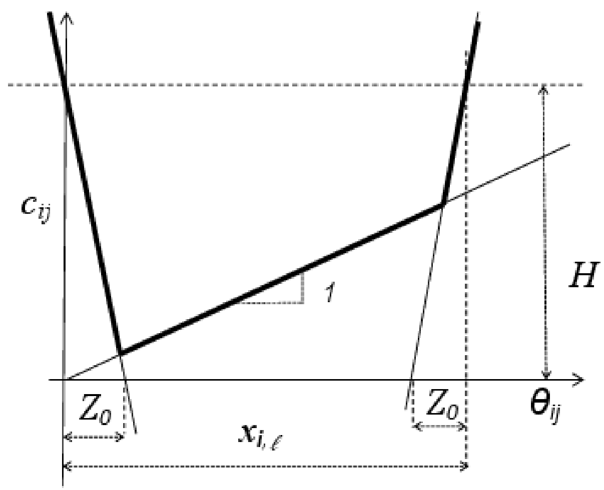

Fig. 2. The cost function (1) for links in A_y and A_a.

The link travel times as functions of the time lags $\delta_{\ell,s}$ will be expressed then as $c(f(\delta))$, where the function $f(\cdot) = (\ldots, f_a(\cdot), \ldots; a \in A_a \cup A_y)$ is defined by:

$$f_a(\delta) = \theta_a = \begin{cases} t_j - t_{i,\ell}^0 - \sum_{\sigma=1}^{s} \delta_{\ell,\sigma}, & a = (i,j) \in A_y(\ell, s) \\ -t_k + t_{j,\ell}^0 + \sum_{\sigma=1}^{s} \delta_{\ell,\sigma}, & a = (j,k) \in A_a(\ell, s) \end{cases} \quad \ell \in L, \quad 1 \leq s \leq n_\ell \tag{4}$$

For the solution method it will be necessary to evaluate the derivatives of the travel time functions vector $C(\delta) \overset{\Delta}{=} c(f(\delta))$ with respect to the time lag vector δ.

$$\mathbf{i} \in \mathbf{S}, \quad \ell \in L(\mathbf{i}), 1 \leq s \leq n_\ell: \tag{5}$$

$$(i,j) \in A_y(\mathbf{i}, \ell, s):$$

$$\frac{\partial C_{ij}}{\partial \delta_{\ell,\sigma}} = \begin{cases} \dfrac{H}{z_0} & if \;\; \theta_{ij} = t_j - t_{i,\ell}^0 - \sum_{\sigma=1}^{s} \delta_{\ell,\sigma} < z_0 \\ \dfrac{-H}{z_0} & if \;\; \theta_{ij} = t_j - t_{i,\ell}^0 - \sum_{\sigma=1}^{s} \delta_{\ell,\sigma} > x_{\mathbf{i},\ell} - z_0 \\ -1 & \text{otherwise} \end{cases}$$

(5')

$$(j,k) \in A_a(\mathbf{i},\ell,s):$$
$$\frac{\partial C_{jk}}{\partial \delta_{\ell,\sigma}} = \begin{cases} -\dfrac{H}{z_0} & if\ \theta_{kj} = -t_k + t_{j,\ell}^0 + \sum_{\sigma=1}^{s} \delta_{\ell,\sigma} < z_0 \\ \dfrac{H}{z_0} & if\ \theta_{kj} = -t_k + t_{j,\ell}^0 + \sum_{\sigma=1}^{s} \delta_{\ell,\sigma} > x_{\mathbf{i},\ell} - z_0 \\ 1 & \text{otherwise} \end{cases}$$

Demand and network flows structure on the diachronic graph

For any o-d pair $\boldsymbol{\omega} = (\mathbf{p},\mathbf{q}) \in \mathbf{W}$ on the planar graph, a minimum o-d travel time $\mathbf{u}_{\boldsymbol{\omega}}$ can be calculated. These minimum travel times are calculated without taking into account transfer times from different lines at stations. Correspondingly let $\lambda_{\boldsymbol{\omega}}^0$ be the minimum number of time ticks to reach destination $\mathbf{q}$ from origin $\mathbf{p}$, i.e., $\lambda_{\boldsymbol{\omega}}^0 = \lceil \mathbf{u}_{\boldsymbol{\omega}}/\Delta \rceil$. Then, the model assumes that the total number of trips originating at $\mathbf{p}$ at time subinterval ν with destination $\mathbf{q}$, $g_\nu^{\boldsymbol{\omega}}$, are known. Because the capacity limitation at links in A_e will be active reflecting congestion effects, the flow $g_\nu^{\boldsymbol{\omega}}$ will divide into flows $g_{\nu\mu}^{\boldsymbol{\omega}}$, arriving at destination $\mathbf{q}$ at several time instants $\mu \geq \nu + \lambda_{\boldsymbol{\omega}}^0$,

$$g_\nu^{\boldsymbol{\omega}} = \sum_{\mu \geq \nu + \lambda_{\boldsymbol{\omega}}^0} g_{\nu\mu}^{\boldsymbol{\omega}}, \quad g_{\nu\mu}^{\boldsymbol{\omega}} \geq 0 \tag{6}$$

Thus, newly defined sets of nodes and links will be added to the diachronic network so that flows $g_{\nu\mu}^{\boldsymbol{\omega}}$ are taken into account properly. For each destination $\mathbf{q} \in \mathbf{D}$ of the planar graph a new artificial node $q(\mathbf{q})$ will be considered where the balance equations (6) will hold. These artificial nodes are the head nodes of links originating at $N_T(\mathbf{q})$. Let $A_g(\mathbf{q}), \mathbf{q} \in \mathbf{D}$ be the set of such links and let $A_g = \cup_{\mathbf{q}\in\mathbf{D}} A_g(\mathbf{q})$. Figure 3 illustrates the previous concepts in the demand structure. Costs for links in A_g are zero. Flows on the diachronic network will be structured in commodities $(\boldsymbol{\omega},\nu) \in \mathbf{W} \times \{1,\ldots,\lceil H/\Delta \rceil\}$ which can be grouped by origins, i.e., if $\boldsymbol{\omega} = (\mathbf{p},\mathbf{q})$, the commodities will be then $(\mathbf{p},\nu) \in \mathbf{O} \times \{1,\ldots,\lceil H/\Delta \rceil\}$, or equivalently flows $g_\nu^{\mathbf{p}} = \sum_{\mathbf{q}\in\mathbf{D}(\mathbf{p})} g_\nu^{\mathbf{p},\mathbf{q}}$ that originate at node $i \in N_T(\mathbf{p})$ for time subinterval ν-th and arrive to artificial nodes $q(\mathbf{q}), \mathbf{q} \in \mathbf{D}(\mathbf{p})$.

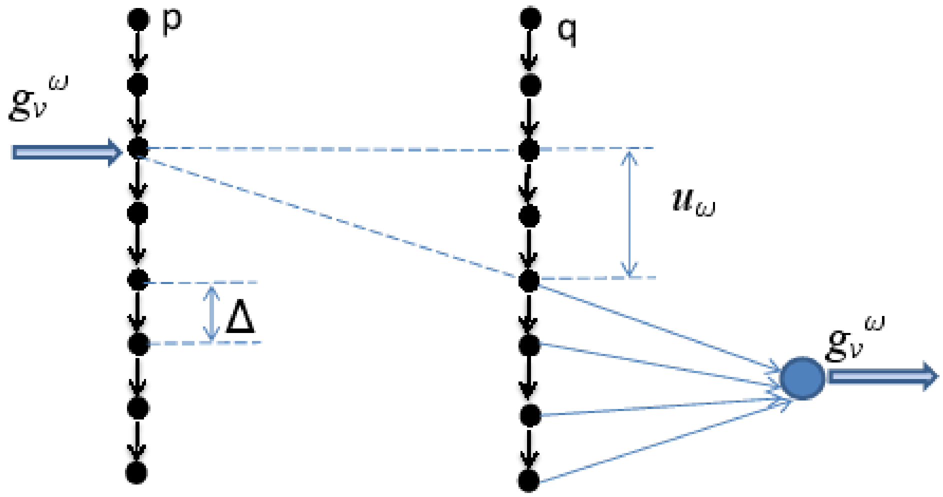

Fig. 3. Schematic representation of the origin-destination flows structure in the diachronic graph.

The network flows problem

Let now $\gamma^{\mathbf{p},\nu} \in \Re^{|N|}$, $\mathbf{p} \in \mathbf{O}$, $1 \leq \nu \leq n$ be a vector defined as:

$$(\gamma^{\mathbf{p},\nu})_i = \begin{cases} g_\nu^{\mathbf{p}} & \text{if } i \in N_T(\mathbf{p}) \text{ for the time subinterval } \nu, \\ -g_\nu^{\mathbf{p}} & \text{if } i = q(\mathbf{q}), \quad \exists \mathbf{q}, \in \mathbf{D}(\mathbf{p}) \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

Let now $\mathrm{v}^{\mathbf{p},\nu}$ be an arc flow vector defined on the diachronic network for links in $A \backslash A_g$, let $\mathrm{v} = \sum_{\mathbf{p},\nu} v^{\mathbf{p},\nu}$ the vector of total flows on links in $A \backslash A_g$, let v_a, $a \in A \backslash A_g$ denote a component of v and let $\mathrm{g}^{\mathbf{p},\nu} = (\ldots, g_\nu^{\mathbf{p},\mathbf{q}}, \ldots; \mathbf{q} \in \mathbf{D}(\mathbf{p}))$ be a vector for o-d demands associated to links in A_g.

If the time lags $\delta_{\ell,s}$, $\ell \in L$, $1 \leq s \leq n_\ell$ are fixed, then the network flow problem determining passenger flows on the time expanded network will be:

$$\begin{array}{llll} \phi(\delta) = Min_{\mathbf{v},g} & C(\delta)^T \mathbf{v} & & \\ s.t.: & B_v v^{p,\nu} + B_g g^{p,\nu} = \gamma^{p,\nu}, & (p,\nu) \in O \times \{1, \ldots, \lceil H/\Delta \rceil\} & \\ & v_a \leq b_\ell, & a \in A_e(\ell), \ell \in \tilde{L} & \\ & v_a \leq v_{a'} \psi_a(f_a(\delta)) \mid \mu_a^B & a = (i,j) \in A_a, & a' \in E(j) \\ & v_a \leq v_{a'} \psi_a(f_a(\delta)) \mid \mu_a^A & a = (j,k) \in A_y, & a' \in I(j) \\ & v^{p,\nu} \geq 0, \quad g^{p,\nu} \geq 0 & & \end{array} \tag{8}$$

where $C(\cdot)$ is the links travel time function vector as a function of the time lags vector δ, that applies for links in $A \backslash A_g$. b_ℓ, is the capacity or maximum number of passengers in a bus operating in a line $\ell \in \tilde{L}$ corresponding to a in-vehicle link $a \in A_e(\ell)$.

Lagrange multipliers μ_a^A, μ_a^B corresponding to constraints $v_a \leq v_{a'}\psi(f_a(\delta))$, appear separated by a bar, "|", from them. B_v, B_g are node-arc incidence matrices for the diachronic network.

The functions ψ_a for boarding and alighting links $a \in A_a \cup A_y$ are defined accordingly to the ratio $x_{i(i),\ell}/\Delta$ which will be assumed an integer $v_{i(i),\ell} \geq 2$.

$$\psi_a(\alpha) = \frac{1}{v_{i(i),\ell} - 1} max\{\, 0, min\{\frac{\alpha}{x_{i(i),\ell}}, 1, (1 - \frac{\alpha}{x_{i(i),\ell}})\}\, \}, \qquad \begin{array}{l} a = (i,j) \in A_y(\ell, s) \\ \ell \in L \end{array} \tag{9}$$

and a similar expression would apply for links $a \in A_a$. Notice that $\psi_a(\alpha) = \psi_a(\alpha + \Delta) = \ldots = \psi(\alpha + (v-1)\Delta)$ and that $\sum_{p=0}^{v-1} \psi_a(\alpha + p\Delta) = 1$, for $0 \leq \alpha \leq \Delta$.

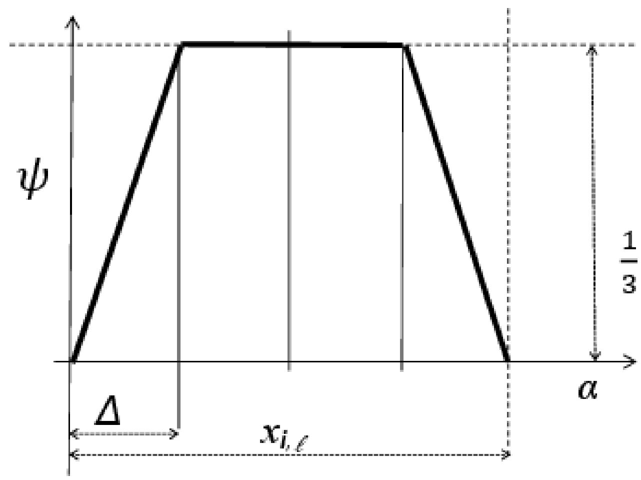

Fig. 4. The function ψ in (9) for $v = 4$.

Function $\phi(\delta)$, the value function of previous problem (8), is non-differentiable and non-convex. In order to minimize it using an optimization algorithm, its subgradients should be evaluated. By using results in Gauvin and Dubeau (1982), at a differentiable point, the partial derivatives of ϕ can be evaluated by:

$$\frac{\partial \phi}{\partial \delta_{\ell,\sigma}} = \frac{\partial}{\partial \delta_{\ell,\sigma}} \left(C(\delta)^T v^* - \sum_{\substack{a \in A_a \\ a' \in E(j(a))}} \mu_a^B (v_{a'}^* \psi_a(f_a(\delta)) - v_a^*) - \sum_{\substack{a \in A_y \\ a' \in I(i(a))}} \mu_a^A (v_{a'}^* \psi_a(f_a(\delta)) - v_a^*) \right) \tag{10}$$

where $\mathbf{v}^*$ is a vector of total link flows solving problem (8), then at points where ϕ is differentiable, $\phi\delta_{\ell,s}$ can be expressed more explicitly as:

$$\frac{\partial \phi}{\partial \delta_{\ell,\sigma}} = \sum_{i \in S} \left\{ \sum_{\substack{a \in A_y(i) \\ a' \in I(i(a))}} (v_a^* \frac{\partial C_{ij}}{\partial \delta_{\ell,\sigma}} + v_{a'}^* \psi'_a + \sum_{\substack{a \in A_a(i) \\ a' \in E(j(a))}} \left(v_a^* \frac{\partial C_{ij}}{\partial \delta_{\ell,\sigma}} - v_{a'}^* \psi'_a \right) \right\}, \quad \ell \in L, s \leq n_\ell \tag{11}$$

where $\psi'_a = \frac{\mathrm{d}\psi}{\mathrm{d}\alpha}(f_a(\delta))$.

A method for evaluating optimal time lags between services

A convenient way of finding the optimal time lags is by solving the following problem:

$$\begin{aligned} Min_\delta \quad & \phi(\delta) \\ s.t.: \quad & \sum_{\sigma=1}^{n_\ell+1} \delta_{\ell,\sigma} = H, \qquad \ell \in L \\ & \delta_{\ell,\sigma} \geq \hat{x}_\ell, \qquad \ell \in L \end{aligned} \tag{12}$$

where $\hat{x}_\ell = max\{x_{\mathbf{i},\ell}; \mathbf{i} \in \mathbf{S}(\ell)\}$ and the additional time lag $\delta_{n_\ell+1}$ can be considered simply as a slack variable. The solution method can be considered a combination of a (sub)gradient projection method and a pattern search. At each iteration, a direction $d' \in \partial\phi(\delta)$ is calculated after solving problem (8) and applying then formula (11). Let $A^+(\delta_\ell)$ be the set of inactive constraints, i.e., $A^+(\delta_\ell) = \{ 1 \leq s \leq n_\ell \mid \delta_{\ell s} > \hat{x}_\ell \}$. Then, its negative projected direction, $\tilde{d}_\ell$, is given by $\tilde{d}_{\ell s} = \bar{d}'_\ell - d'_{\ell s}$ if $s \in A^+(\delta_\ell)$ and $\tilde{d}_{\ell s} = 0$ if $s \notin A^+(\delta_\ell)$, where $\bar{d}'_\ell = \frac{1}{n_\ell}\sum_s d'_{\ell s}$ is the mean value of the components in $d'_\ell, \ell \in L$. A small number of evaluations of ϕ are along direction $\tilde{d}$ are performed with step lengths α bounded by $\alpha_0 \leq \alpha \leq \alpha_1$, provided that $\phi(\delta) > \phi(\delta + \alpha_0\tilde{d})$. The projected direction is considered to fail if $\phi(\delta) \leq \phi(\delta + \alpha_0\tilde{d})$ and $\|\tilde{d}\| \geq \varepsilon$. Bounds α_0, α_1 are established so that constraints on the time lags $\delta_{\ell s}$ are not violated and their fluctuations are at least Δ, the time granularity of the model. If the projected direction fails, then the opposite direction $-\tilde{d}$ is searched using the same procedure with suitable bounds $\alpha_{0'}$ and $\alpha_{1'}$. In case that a new failure in getting lower values for ϕ happens, then an adaptation to the problem of the pattern search method in Lewis and Torczon (2000) is initiated. The objective function ϕ is evaluated at points on the opposed orthant containing $\tilde{d}$. Thus, it is evaluated at a sequence of points $\delta_\ell^{(k} + \mu_0\tilde{e}_s$, where $\tilde{e}_s$ is the projected direction of $e_s \in \Re^{n_\ell}$ which is given by:

$$(e_s)_\sigma = \begin{cases} \pm\tilde{d}_{\ell,s}/|\tilde{d}_{\ell,s}|, & \text{if } \sigma = s \text{ and } s \in A^+(\delta_\ell^{(k}) \\ 0 & \text{otherwise.} \end{cases} \quad (13)$$

where step length $\mu_0 = n_0\Delta$ if $\delta_\ell^{(k} + \mu_0\tilde{e}_s \geq \hat{x}_\ell$ or $\mu_0 = 0$ otherwise and the movement will be discarded. If a movement $\tilde{e}_s$ is found so that $\phi(\delta_\ell^{(k} + \mu_0\tilde{e}_s) < \phi(\delta^{(k)})$ then $\delta_\ell^{(k+1} = \delta_\ell^{(k} + \mu_0\tilde{e}_s$. Otherwise it is considered that no enhancement can be done and the algorithm stops.

Due to the simplifications of the model, after finding an approximate solution to problem (9) the following aspects should be inspected: a) it should be checked whether the operational times at stops $x_{i,\ell}$ are consistent with the total boarding and alighting flows v_a, $a \in A_y$, $a \in A_a$ so that these operations have a reasonable time to be performed. Also, the overlapping of arrivals at stops from transportation units of different lines, if it is the case, should be solved satisfactorily.

Running the algorithm

The model described in previous sections and the algorithm for solving it have been implemented in AMPL and tested on the small test network shown in figure 5 below using CPLEX 12.5. The tests have been carried out on a working station R5500 with processor Intel(R) Xeon(R) CPU E5645 2.40 GHz and 48 Gbytes of RAM. On this network six lines have been defined each of them with 6 services on a time span of 360 time subintervals of 0.5 minutes each. Each service corresponds to a bus run with capacity for 50 passengers. The following origin destination pairs have been defined $W = \{(10,14)(10,20)(10,24)(14,20)(20,24)\}$, where input flows at origins has been set to 1 passenger each 0.5 minutes (i.e., per time subinterval) for each destination. The six lines are the following ones: line L1 starts at node 10, stops at nodes 12, 17, 22 and ends at node 24. Line L2 starts at node 24, stops at nodes 22, 17, 12 and ends at node 10. line L3 starts at node 10 and then stops at 15, 15, 17, 19, and ends at node 24. Line L4 starts at node 24, stops at nodes 19, 17, 15 and ends at node 10. Line L5 starts at node 14, stops at nodes 19, 17, 15 and ends at node 20. Line L6 starts at node 20, stops at nodes 15, 17, 19 and ends at node 14. Dwell time at stops has been fixed at $x = 1min$. Each link in the network is 500m long, speed of buses is assumed to be 35 km/h and pedestrian's speed is assumed 3 km/h. The resulting diachronic network has 3450 nodes and 6551 links.

Because the heuristic nature of the problem mild stopping tolerances have been used, that have been reached after a number of iterations ranging from 11 to 15. In none of the runs was necessary the pattern search method in order to overcome a failing descent projected direction. Solving instance of problem 8 has taken an average of 35 seconds approximately.

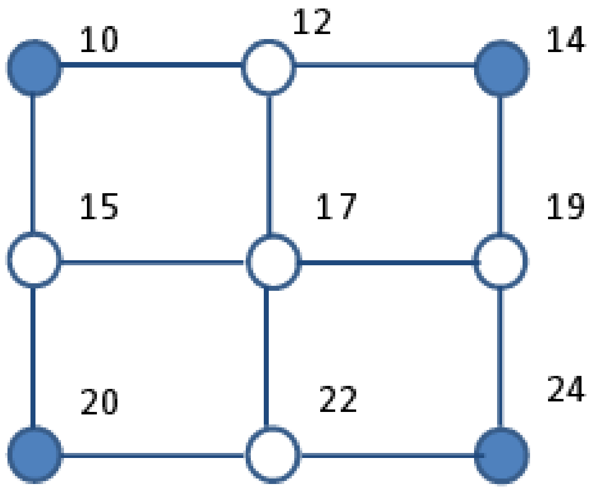

Fig. 5. A small test network.

Conclusions

A model for scheduling services on public transportation lines has been developed that is capable of handling dynamic variations of the demand with an objective of minimizing the total passenger's travel time. The optimization of the schedules is done using an adaptation of the projected gradient algorithm in order to minimize the total travel time. A diachronic network model is developed on which linear programming problems are solved. Despite the large size of the diachronic network, tests on small size networks show that the linear programming problems can be solved efficiently showing that it can be used either for offline planning using a large period of 2 to 3 hours or, in an online optimization context using shorter periods of ~ 30 minutes.

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