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A closed-loop supply chain repackaging linear optimization problem

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Abstract. This paper presents a model for optimizing the costs of a complex production, distribution and package reprocessing industry. Building on previous work, closed-loop supply chain activities are incorporated in the form of a reverse flow of empty packages and reprocessing of those packages for further utilization. A linear programming mathematical framework is proposed for representing the different elements of the industry, allowing for high flexibility when applying the model to different industries. The optimization model provides the optimal decision variables for inventory management, production and recycling planning and scheduling, transport routing and empty package purchasing. The proposed framework allows for the introduction and alteration of multiple parameters in each of the different elements of the chain. Finally, a numerical example, involving a bottling industry that deals with standardized packaging, is solved utilizing Gurobi software, and a minimal cost solution is found, thus proving the model's validity.

Keywords: closed-loop supply chain; linear programming; optimization; bottling; scheduling

Introduction

In recent years, there has been increasing interest in reverse flows and closed-loop supply chain elements in most industries. As environmental concerns have grown, it has become necessary to incorporate new activities such as waste disposal, gathering and reprocessing. More specifically, in the bottling industry, special attention has been paid to the reutilization of used containers. However, these activities carry additional costs and particularities that can affect the entirety of chain logistics and profitability. Our objective in this work is to expand an existing mathematical framework by incorporating reverse activities and subsequently perform a linear optimization in order to minimize the global total cost. The whole industry is modeled with great flexibility allowing our framework to be easily modified and applied for optimizing similar industries.

When it comes to the reverse flow, our model considers a case of package reprocessing and reutilization. Through the use of standardized packaging, we take into account a situation in which used packages (herein, bottles) are gathered and reprocessed in order to be filled with the same product, or alternatively with different final products that share the same standardized bottle. This adds new complications when it comes to inventory management and production scheduling. We consider a deterministic approach, given that we work with a planning horizon in mind.

The main characteristic of our work is that it considers the entire industry situation as a whole. While most analyses are normally performed in order to optimize one of the different elements of the business, our cost optimization framework takes into account how inventory, production and distribution decisions affect each other, considering that all elements are interrelated. By doing so, a more efficient solution can be found, calculating optimal inventory amounts, and production and recycling planning variables, such as the order and amount of products that have to be transported at each time to each receiver. However, this whole approach also means that our final objective function will feature an even bigger number of variables, and it will be tightened by more constraints. This requires our parameters to be chosen carefully in order to allow a feasible solution to be found.

Literature review

The main influence on this paper is a study developed by Kopanos, Puigjaner *et al* (2011). The authors proposed a mathematical framework representing the multiple operations of a dairy product industry in Greece by incorporating the particularities of dairy products into production scheduling and performing a computational cost optimization. Our aim is to expand this proposed framework to a different industry, adding at the same time closed-supply chain activities in the form of package reprocessing.

As one of the more prevalent topics discussed in this article, closed-supply chains used in order to process and reuse packages have been extensively discussed in the academic literature. Chuang, Wee *et al* (2007) analyzed an inventory system with both forward and reverse material flow. In their model, products belonging to the reverse flow are remanufactured and sent to the retailer so that they could be sold again. Silva, Reno, Sevegnani *et al* (2012) performed a study into the benefits and drawbacks of using returnable packaging in a closed loop supply chain comparing it with a previous real world situation whereby disposable packaging was used. Given how recent the general appearance of reverse logistics in the Supply Chain literature is, some studies have been carried out in order to gather the main concepts and analyze already-existing knowledge. One such study was performed by Fleishmann, Bloemhof-Ruwaard *et al* (1997). In their literature review they proposed a series of frameworks for integrating the field of reverse logistics into numerical problems, reviewing previously-used models and proposing further expansions.

Industry Structure

In this paper, we aim to minimize the total cost structure of a bottling (packaging) industry by optimizing at the same time the transportation operations, inventory management and production scheduling. The main characteristics of this model are:

- A number of bottling plants and facilities addressed by the subindex $s \in S$, with being S the total set of plants.
- There are a number of distribution centers $d \in D$ to be supplied by the plants.
- The set S_d defines which plants can supply each of the distribution centers.
- The plants bottle different products p in bottles of different type t. Each product p can be filled into one type of bottle, but each type of bottle can be used for more than one product p. Which product is filled into each type of bottle is defined by the set P_t.
- Each of the distribution centers in each time period has a demand ζ_{dpn} for each filled product p.
- At the same time, each distribution center has a return supply of dirty, empty bottles (containers) defined by ζ^D_{dtn}.
- At the beginning of each time period, one truck l can transport products between one plant and one DC (distribution center) only once. If the truck goes to a DC in one time period, it will also make the return journey on the same day, transporting if necessary dirty, empty bottles.
- Each plant has three different inventories of products and bottles (figure 1). The inventories are calculated at the end of each time period.
- I^F_{spn} represents the inventory of filled bottles of product p that passed the quality test and are ready to be sent at any moment to supply demand.
- I^R_{spn} represents the inventory of reprocessed bottles of type t that have been cleaned and are prepared for being filled again.
- I^D_{stn} represents the inventory of dirty bottles of type t that have to be reprocessed in order to be used again.
- At each plant, there is a number of production lines j that can either fill bottles or reprocess them. The set J_p contains the lines that can fill bottles of product p while J^P_t contains those lines depending on the type of bottle that can be processed, given our package standardization. This means that if a line can fill bottles of product p, it will be included as a line that can fill bottles of type t when $p \in P_t$. The set J^R_t includes the lines that can recycle and clean bottles of type t. It should be noticed that one line can only do one task of either filling or reprocessing, therefore $J^R_t \cap J^P_t = \nexists$. Whether each plant can use each of the production lines is defined in J_s.
- When it comes to recycling and reprocessing, each type of bottle requires a preparation time and can be processed at a different rate depending on the line and the plant. In the case of the processing lines for bottling, all products belonging to the same type of bottle are processed consecutively, each of them requiring its own preparation time and featuring a different bottling rate.

- In all lines, when more than one type of bottle is processed in the same line in the same time period, consideration has to be made for an exchange time required for adapting the machines to the new type of bottle. This change also involves an additional cost. Both the cost and the time are parameters depending on the order in which the change is made. The optimal order will be obtained in the model solution as one of the decision variables.
- In conclusion, the main decision variables will be: the amount of bottles filled and reprocessed in each time period; the trucks that will satisfy the DC's demands; the storage amount on each of the three inventories; the production lines used in each time period for each of the processes; the purchased amount of new bottles; the order, starting times and finishing times of the filling and recycling operations for each product and type of bottle, respectively.

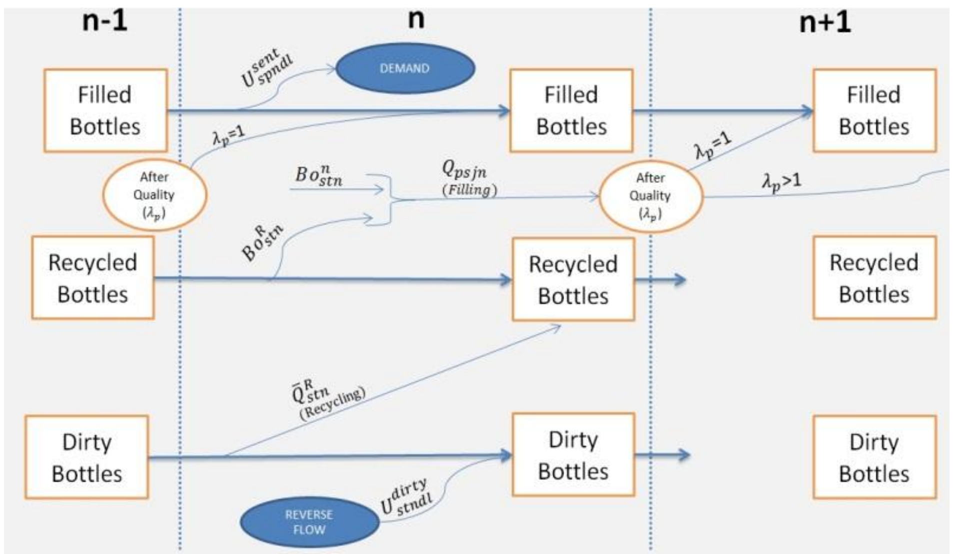

Fig. 1. Each plant with three different inventories of products and bottles

Mathematical Formulation

In this section we will present the main constraints of our linear programming model as well as the final objective function. The following constraints represent the inventory management for filled, recycled and dirty bottles as well as the transportation constraints.

$$I^F_{spn} = I^F_{sp(n-1)} + \sum_{j \in (J_s \cap J_p)} Q_{psj(n-\lambda_p)} - \sum_{d \in D_s} \sum_{l \in L_{sd}} U^s_{spndl}$$

$$\forall s, p, n > \lambda_p$$

$$\sum_{p \in P_T} \sum_{j \in (J_s \cap J_P)} Q_{psjn} = Bo^n_{stn} + Bo^r_{stn} \qquad \forall s, t, n$$

$$Bo^R_{stn} \le I^R_{st(n-1)} \qquad \forall s, t, n$$

$$\overline{Q}^R_{stn} \le I^D_{st(n-1)} \qquad \forall s, t, n$$

$$\sum_{d \in D_s} \sum_{l \in L_{sd}} U^s_{sdlpn} \le I^F_{sp(n-1)} \qquad \forall s, p, n$$

$$I^R_{stn} = I^R_{st(n-1)} + \overline{Q}^R_{stn} - Bo^R_{stn} \qquad \forall s, t, n$$

$$I^D_{stn} = I^D_{st(n-1)} - \overline{Q}^R_{stn} + \sum_{d \in D_s} \sum_{l \in L_{sd}} U^d_{stndl} \qquad \forall s, t, n$$

where λ_p represents the time, counted as the number of time periods, necessary to perform a quality analysis after each product is bottled. Q_{psjn} represents the amount of products bottled on each production line of each plant while U^S_{spndl} stands for the transported products from each plant to each distribution center. In the case of recycled bottles, Q^R_{stn} is the amount of bottles reprocessed and cleaned on each plant on each period. Bo^R_{stn} represents the amount of empty clean bottles that are refilled on each time period. When it comes to dirty bottles, U^D_{stndl} is the reverse flow of empty bottles from each distribution center. Since the production and transportation operations start at the beginning of each time period, and the inventories are counted at the end of each time period, we have to consider some logic constraints.

$$\overline{U}^s_{sdln} = \sum_p U^s_{sdlpn} \qquad \forall s, d \in D_s, l \in L_{sd}, n$$

$$\overline{U}^d_{sdln} = \sum_t U^s_{sdltn} \qquad \forall s, d \in D_s, l \in L_{sd}, n$$

$$\sum_{s \in S_d} \sum_{l \in L_{sd}} U^s_{sdlpn} = \zeta_{dpn} \qquad \forall d, p, n$$

$$\sum_{s \in S_d} \sum_{l \in L_{sd}} U^d_{sdltn} = \zeta^D_{dtn} \qquad \forall d, t, n$$

$$\epsilon_l^{min} Z_{sdln} \leq \overline{U}_{sdln}^{s} \leq \epsilon_l^{max} Z_{sdln}$$
$$\forall s, d \in D_s, l \in L_{sd}, n$$

$$\epsilon_l^{min} Z_{sdln} \leq \overline{U}_{sdln}^{d} \leq \epsilon_l^{max} Z_{sdln}$$
$$\forall s, d \in D_s, l \in L_{sd}, n$$

$$\sum_{s \in S_l} \sum_{d \in (D_s \cap D_l)} Z_{sdln} \leq 1 \qquad \forall l, n$$

The transported amount by each truck on each time period between a plant and a distribution center has to be equal to the sum of the amounts of all the products or empty bottles transported in both the forward and reverse flow respectively. The trucks have minimum and maximum capacity limits. It should be noticed that Z_{sdln} is a binary variable that represents whether the truck will be used for a certain route on a certain period. In our model we assume that one truck can only do one route in each time period. As expected, product demand has to be satisfied. At the same time, there is a dirty bottle return amount on each distribution center that has to be transported on the reverse flow.

The next sets of constraints represent the production operations for both bottling and recycling and incorporate some decision variables that will allow us to obtain a production scheduling solution. The first set represents the total production limits for each product or empty bottle on each line, where $\overline{Y}_{psjn}$ and $\overline{Y}_{tsjn}$ are binary variables associated to whether each production line is being used for each process or not.

$$\pi_{psjn}^{min} \overline{Y}_{psjn} \leq Q_{psjn} \leq \pi_{psjn}^{max} \overline{Y}_{psjn}$$
$$\forall p, s, j \in (J_s \cap J_p), n$$
$$\pi_{tsjn}^{min} Y_{tsjn} \leq Q_{tsjn}^{R} \leq \pi_{tsjn}^{max} Y_{tsjn}$$
$$\forall t, s, j \in (J_s \cap J_t^R), n$$

The next constraints include the definition of total processing time T of each type of bottle in both the filling and recycling operations. In the case of filling this time is equal to the sum of all the preparation times δ_{psj} of each product using the same type of bottle plus the processing time of that product, obtained by dividing the amount produced by the processing rate ρ_{psj}. In the case of the recycling and cleaning lines, it is simpler, due to the processing time depending only on the type of bottle and not the product being bottled. We have to include some logical constraints relating to the bottle type allocation variables. They specify that if a product that uses a type of bottle is being processed in a line at some moment, that type of bottle must be considered as being processed on that line. This only applies for the bottling lines.

$$T_{tsjn} = \frac{Q^R_{tsjn}}{\rho^R_{tsj}} + \delta^R_{tsj} Y_{tsjn}$$
$$\forall t, s, j \in (J_s \cap J^R_t), n$$

$$T_{tsjn} = \sum_{p \in P_t} \left(\frac{Q_{psjn}}{\rho_{psj}} + \delta_{psj} \overline{Y}_{psjn}\right)$$
$$\forall t, s, j \in (J_s \cap J^P_t), n$$

$$Y_{tsjn} \leq \sum_{p \in P_t} \overline{Y}_{psjn} \qquad \forall t, s, j \in (J_s \cap J^P_t), n$$

$$\overline{Y}_{psjn} \leq Y_{tsjn} \qquad \forall t, p \in P_t, s, j \in (J_s \cap J^P_t), n$$

One of the decision variables of our model represents the order in which we should process the different types of bottles in order to minimize the associated change cost in both the recycling and bottling lines. This change also takes some time, which must be taken into account. $X_{tt'sjn}$ is a binary variable equals to 1 if a bottle t is processed before t'. The following constraints limit the types that can be processed before and after t to only one. The binary variable V_{sjn} is equal to 1 if a production line on a plant will be used at all or not in one specific time period, depending on whether a product is being bottled into it or that type of bottle is being reprocessed.

$$\sum_{t \in T_j} \sum_{t' \neq t, t' \in T_j} X_{tt'sjn} + V_{sjn} = \sum_{t \in T_j} Y_{tsjn}$$
$$\forall s, j \in ((J^P_t \cup J^R_t) \cap J_s), n$$

$$V_{sjn} \geq Y_{tsjn} \qquad \forall t, s, j \in ((J^P_t \cup J^R_t) \cap J_s), n$$

$$\sum_{t' \neq t, t' \in T_j} X_{t'tsjn} \leq Y_{tsjn}$$
$$\forall t, s, j \in ((J^P_t \cup J^R_t) \cap J_s), n$$

$$\sum_{t' \neq t, t' \in T_j} X_{tt'sjn} \leq Y_{tsjn}$$
$$\forall t, s, j \in ((J^P_t \cup J^R_t) \cap J_s), n$$

The last set of equations represent the production time limits on each production line, constrained by the parameters that represent the total available time ϖ_{sjn}, line closing time β_{sjn} and line opening time α_{sjn}, as well as the already-mentioned changeover times. At the same time a new decision variable C_{tsjn} is introduced representing the finishing time of bottling or recycling each type of bottle on each line. This variable is part of our production scheduling planning solution.

$$C_{tsjn} + \gamma_{tt'sj} \leq C_{t'sjn} - T_{t'sjn} + (\omega_{sjn} - \beta_{sjn})(1 - X_{tt'sjn})$$

$$\forall s, t, t' \neq t, j \in (((J_t^P \cap J_{t'}^P) \cup (J_t^R \cap J_{t'}^R)) \cap J_s), n$$

$$C_{tsjn} - T_{tsjn} \geq \alpha_{sjn} Y_{tsjn} + \sum_{t' \neq t, t' \in T_j} \gamma_{t'tsj} X_{t'tsjn}$$

$$\forall s, t, j \in ((J_t^P \cup J_t^R) \cap J_s), n$$

$$C_{tsjn} \leq (\omega_{sjn} - \beta_{sjn}) Y_{tsjn}$$
$$\forall s, t, j \in ((J_t^P \cup J_t^R) \cap J_s), n$$

The objective function minimizes the total costs of all operations. In the first place we have the cost for all 3 inventories of filled, cleaned and dirty bottles multiplied by their unitary costs per time period (ζ_{spn}^F,ζ_{spn}^R, ζ_{spn}^D). We consider also the cost per time unit of filling (θ_{psjn}) and reprocessing (θ_{psjn}^R) operations and multiply it for the amount processed divided by the processing rate, this being the processing time. In addition, there are additional fixed costs (ν_{sjn}) depending on whether a line is used or not every time period and a cost ($\phi_{tt'sjn}$) involved in the bottle change operations within the same line. Lastly, we have to consider the fixed (ψ_{sl}) and variable cost (υ_{sdl}) of transportation items in both the forward and reverse flow as well as the purchase unitary cost (ϑ_{stn}^n) of new bottles multiplied by the amount of bottles purchased in each time period.

$$\begin{aligned}
Minimize \quad & \sum_s \sum_p \sum_n \zeta_{spn}^F I_{spn}^F + \sum_s \sum_t \sum_n \zeta_{stn}^R I_{stn}^R \\
& + \sum_s \sum_t \sum_n \zeta_{stn}^D I_{stn}^D + \sum_s \sum_p \sum_{j \in (J_p \cap J_s)} \sum_n \frac{\theta_{psjn}}{\rho_{psj}} Q_{psjn} \\
& + \sum_s \sum_t \sum_{j \in (J_t^R \cap J_s)} \sum_n \frac{\theta_{psjn}^R}{\rho_{psj}^R} Q_{psjn}^R + \sum_s \sum_{j \in J_s} \sum_n \nu_{sjn} V_{sjn} \\
& + \sum_s \sum_t \sum_{t' \neq t} \sum_{j \in (((J_t^P \cap J_{t'}^P) \cup (J_t^R \cap J_{t'}^R)) \cap J_s)} \sum_n \phi_{tt'sjn} X_{tt'sjn} \\
& + \sum_s \sum_t \sum_n Bo_{stn}^n \vartheta_{stn}^n + \sum_s \sum_{d \in D_s} \sum_{l \in L_{sd}} \sum_n (\psi_{sl} Z_{sdln} + \upsilon_{sdl} (\overline{U}_{sdln} + \overline{U}_{sdln}^d))
\end{aligned}$$

Numerical Solving

In order to prove the model's validity we have solved a numerical example with parameters that aim to imitate a real industry. We consider a planning horizon of 4 different time periods n, each of them representing one day. Our proposed business consists of 2 bottling and recycling plants s0 and s1 and 2 distribution centers d0 and d1. The distribution centers have a daily demand for each of the 4 products p0, p1, p2 and p3 which are bottled in the plants in 2 different types of standardized bottles: p0 and p1 are bottled with bottles of type t0 while p2 and p3 require bottles of type t1. At the same time, both of the distribution centers feature a daily return of empty bottles of both types that are transported, collected and reprocessed in the plants for refilling. The demand and return amounts can be found in the following figures.

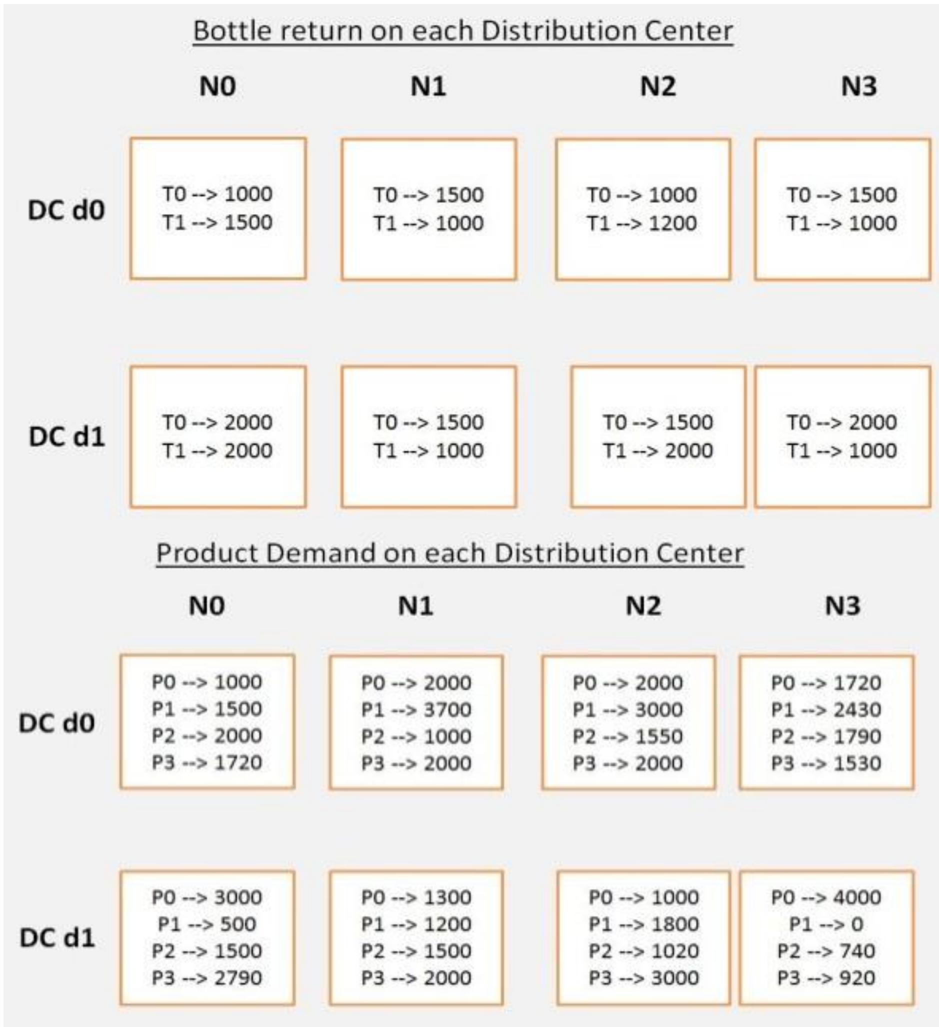

Fig. 2. The demand and return amounts

Trucks l0 and l1 can transport material from and to plant s0 while trucks l2 and l3 do the same with plant s1. All trucks have a maximum capacity of 7000 bottles per day and a minimum of 1000. Inside each of the plants we consider 3 different production lines: j0 with a processing rate ρ of 700 bottles per hour and j1 with a rate of 1000 bottles per hour. These two lines are used for filling operations while j2 is the one in charge of the recycling operations with a processing rate ρ of 600 bottles per hour. The preparation times δ are 0.5 hours for each product and bottle type and the total physical available time ω is 10h for all lines each daily time period, with an opening time α of 1 hour for all lines and a closing time β of 1 hour for the filling lines and 2 hours for the recycling line. Finally, when changing in the same line between different types of bottles, the changeover time required γ on the filling lines is 1 hour, and 0.5 hours for the recycling line. The daily maximum production limits on each line have a value of 4000 units of each product on each filling line and 3000 units of each type of bottle on the recycling line. λ_p is 0 for all products.

With these settings we solved our problem using a CPU Intel i7-3770 with 8GB of RAM utilizing Gurobi Optimizer 5.5. The final optimization problem featured 942 constraints and 652 variables, taking a total computational time of 98.26 seconds. The output of our problem includes all detailed transportation operations in each time period as well as filling, recycling and inventory amounts. The following tables include the optimal transportation amounts results in both the forward U^s and the reverse U^d flow as well as the optimal choices of trucks for each time period and the optimal amount of new bottles purchased in order to minimize the total cost.

Table 1. Transportation amounts

Plant s	Bottle type t	Time period n	Purchased Bottles
0	0	0	200
0	1	0	0
1	0	0	0
1	1	0	0
0	0	1	2070
0	1	1	1510
1	0	1	560
1	1	1	1270
0	0	2	4200
0	1	2	1000
1	0	2	1200
1	1	2	0
0	0	3	0
0	1	3	0
1	0	3	0
1	1	3	0

Table 2. Transportation amounts

Plant s	DC d	Truck l	Time Period n	U^s	U^d
0	0	1	0	6220	2500
0	1	0	0	1490	1000
1	1	3	0	6300	3000
0	0	1	1	7000	1500
1	0	2	1	1700	1000
1	1	3	1	6000	2500
0	0	0	2	1550	1000
0	1	1	2	6820	3500
1	0	3	2	7000	1200
0	0	0	3	2840	1000
0	1	1	3	5660	3000
1	0	3	3	4630	1500

The next tables represent the optimal inventory amounts at the end of each time period for recycled and dirty bottles I^R, I^D and filled products I^F as well as the amount of products and bottles filled and recycled during that time period. Since the inventories are calculated at the end of each time period, we have considered an initial inventory amount of 3000 filled products and 2000 recycled and dirty bottles of each type.

Table 3. Inventory amounts

Plant s	Bottle t	Time period n	I^R	I^D	Recycled bottles
0	0	0	1300	1700	1300
0	1	0	3490	2500	2000
1	0	0	4000	2000	2000
1	1	0	1300	1700	1300
0	0	1	800	1400	800
0	1	1	2500	1000	2500
1	0	1	1820	2680	1820
1	1	1	1480	1220	1480
0	0	2	0	3900	0
0	1	2	0	3000	0
1	0	2	0	2680	0
1	1	2	0	2420	0
0	0	3	0	6900	0
0	1	3	0	4000	0
1	0	3	0	3180	0
1	1	3	0	3420	0

Table 4. Inventory amounts

Plant s	Product p	Time period n	I^F	Filled products
0	0	0	2000	1300
0	1	0	2400	900
0	2	0	810	0
0	3	0	1790	510
1	0	0	1300	0
1	1	0	2500	0
1	2	0	1690	0
1	3	0	2210	2000
0	0	1	1000	1000
0	1	1	2370	2370
0	2	1	1020	1020
0	3	1	3980	3980
1	0	1	2130	2130
1	1	1	2430	2430
1	2	1	1550	1550
1	3	1	1020	1020
0	0	2	4000	4000
0	1	2	1000	1000
0	2	2	1050	1050
0	3	2	2450	2450
1	0	2	1720	1590
1	1	2	1430	1430
1	2	2	1480	1480
1	3	2	0	0
0	0	3	0	0
0	1	3	0	0
0	2	3	0	0
0	3	3	0	0
1	0	3	0	0
1	1	3	0	0
1	2	3	0	0
1	3	3	0	0

With these results, the optimal minimum total cost was calculated to be 16026 dollars. As it can be easily calculated, all the inventory and transportation constraints are validated in the results and an optimal solution is calculated. The cost associated with the purchase of new bottles is minimized owing to the recycling operations and indeed, it would be even lower if we had considered a bigger amount of initial empty and dirty bottles to be recycled.

Conclusion

This paper has extended a previous mathematical framework in order to implement reverse flows and recycling activities in a linear programming cost optimization problem. The study considers a business of bottling and recycling plants, taking into account that it must satisfy a series of demands and at the same time recycle and reuse empty incoming bottles in order to reduce ecological impact and the package purchase cost. In order to prove the model's efficiency, a numerical example has been solved taking into account all the described equations and the full objective function. The output of this example has given detailed information on inventory management, transportation routing, production scheduling, material acquisition and operation placement due to a high number of decision variables. The combined implementation of reverse repackaging logistics with whole-industry optimization constitutes an innovative approach in the closed-loop supply chain field. Further improvement of the model should be focused on considering alternative remanufacturing processes, as well as separate activities and disposal activities. A multi-optimization approach implementing demand peaks should also be considered.

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