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Kalman-Filtering formulation details for Dynamic OD passenger matrix estimation

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Abstract. In this paper, we describe how to estimate time-sliced origin-destination (OD) matrices for passengers in a public transport network based on counts of ICT (Intelligent Communication Technology) devices carried by passengers at equipped transit-stops. The transit assignment framework is based on optimal strategy, which determines the subset of paths related to the optimal strategies between all OD pairs for the whole horizon of study. Details are provided on how to build the involved equations in a linear Kalman filtering model formulation, which is defined by the authors for a toy network that is proposed to validate the approach.

Keywords: matrix estimation; kalman-filtering; advanced transport information systems

Introduction

The estimation of dynamic OD passenger matrices has received little attention in the literature due to the difficulties in collecting real-time passenger data. However, new technologies have generated proposals for automated passenger count (APC) and Automated Data Collection System (ADCS) applications to be used in transport planning, with a focus on the transit Origin-Destination inference. However, OD matrices are not yet directly observable; consequently, it has been natural to resort to indirect estimation methods. Wong and Tong (1998) proposed a maximum entropy estimator that employs the schedule-based approach for dynamic transit assignment. Ren (2007) proposed a generalized least squares bilivel approach for estimating time-dependent passenger matrices in congested schedule-based transit networks that have automatic passenger counts (APC) and prior OD matrices. The proposal was tested in a toy network, but no recent works from the author have confirmed either offline or online applicability to large scale transit networks. Kostakos *et al* (2010) proposed the use of passengers' Bluetooth mobile devices to derive passenger OD matrices in a simplified context.

Our aim is to explore the possibility of making use of this new data to estimate real-time dynamic Origin/Destination matrices of passengers in a transit network, using a sample of equipped passengers provided by data collected from antennas located in a subset of transit stops.

The remainder of this paper is structured as follows. Firstly, we describe the formulation for estimating passenger matrices in public transport. Next, the approach is tested and validated on a toy network. And finally, the conclusions are stated.

Model Formulation

Notation is defined in Table 1. Some aspects of the data model and formulation statement that have to be considered are:

- The demand matrix for the period of study is assumed to be divided into several time-slices, accounting for different proportions of the total number of passengers in the time horizon.
- The approach assumes an extended space state variable for M+1 sequential time intervals of equal length Δt (between 5 and 10 minutes for transit matrices), in order to consider non-instantaneous travel times. M should guarantee traversing the network.
- Bluetooth antennas are ICT sensors assumed to be located at (some) transit-stops.
- OD paths involved in optimal strategies for transit trips in the period of study can be computed by any transportation planning software that includes a strategy-based equilibrium transit assignment for historic demand. We do not have a strategy-based dynamic transit assignment tool available because we faced a delay in the development; so we have used EMME4 (2013) to define state-variables in our tests. To map the OD paths involved in optimal strategies, we used the EMME output to build input files of the MatLab data model, to systematically program in python those paths involved in optimal strategies from centroid *i* to *j* and which pass through a pair *(r,s)* of ICT sensors.

Table 1. Definition of model variables

$\tilde{Q}_i(k)$, $\tilde{q}_i(k)$	:	Historic total number of passengers and ICT equipped passengers accessing a transit unit at any stop inside the transportation area modeled by centroid *i* at time interval *k*.
$Q_i(k)$, $q_i(k)$	:	Total number of passengers and BT equipped passengers accessing a transit unit at any stop inside the transportation area modeled by centroid *i* at time interval *k*.
$\tilde{y}_q(k)$, $y_q(k)$	:	Historic and actual number of equipped passengers crossing ICT sensor *q* or *s* from the pair of sensors *(r,s)=q* at time interval *k*
$G_{ije}(k)$, $\tilde{G}_{ije}(k)$, $g_{ije}(k)$, $\tilde{g}_{ije}(k)$	:	Total number of current $G_{ije}(k)$ and historic $\tilde{G}_{ije}(k)$ passengers as well as current $g_{ije}(k)$ and historic $\tilde{g}_{ije}(k)$ equipped passengers accessing centroid *i* at time interval *k* and headed towards *j* using path *e*.

$\Delta g_{ije}(k)$	:	State variables are deviates of equipped passengers accessing centroid *i* during interval *k* and headed towards centroid *j* using path *e*, with respect to historic data $\Delta g_{ije}(k) = g_{ije}(k) - \tilde{g}_{ije}(k)$.
$z(k)$, $\tilde{z}(k)$	:	The current and historic measurements of equipped passengers during interval *k*, a column vector of dimension Q, plus balance equations.
$u_{rs}^{h}(k)$	:	Fraction of equipped passengers that require *h* time intervals to reach ICT sensor in transit stop *s* at time interval *k* from ICT sensor *r*. Time-varying model parameters. The values of $u_{rs}^{h}(k)$ are updated according to the measurements of the ICT sensors
$u_{ije,rs}^{h}(k)$	:	Fraction of equipped passengers detected at interval k whose trip from centroid *i* to sensor *s from sensor r* might use the OD path *e* and that last h time intervals of length Δt, $h = 1 \ldots M$.
$\bar{t}_{rs}(k)$	:	Average measured travel time for equipped passengers captured by the pair of ICT sensors *(r,s)* crossing sensor *s* during interval *k*

The total number of origin centroids (related to transportation zones) is I, identified by index *i*, from *1* to *I*; the total number of ICT sensors is P and the considered pairs of ICT sensors *(r,s)* is Q, where P ICT sensors are located either at bus-stops or at segments in the inner network; and the total number of paths (*K*) corresponding to optimal (static) transit strategies from the historic OD transit matrix for the period of study. Each equipped transit stop could be considered either an origin or a destination, and it models a transit-stop that might be shared by several transit lines; but we prefer to estimate OD transit trips between OD pairs, not between transit-stops.

The state variables $\Delta g_{ije}(k)$ are assumed to be stochastic in nature. An autoregressive model of order r <<M is used to relate OD path flow deviations at the current time *k* to the OD path flow deviations of previous time intervals. The state equations are:

$$\mathbf{\Delta g}(k+1) = \sum_{w=1}^{r} \mathbf{D}(w)\mathbf{\Delta g}(k-w+1) + \mathbf{w}(k), \tag{1}$$

where $\mathbf{w}(k)$ has zero mean with a diagonal covariance matrix $\mathbf{W_k}$, and $\mathbf{D}(w)$ are transition matrices which describe the effects of previous OD flows $\mathbf{\Delta g_{ije}}(k-w+1)$ on current flows $\mathbf{\Delta g_{ije}}(k+1)$ for $w = 1,\ldots,r$. If no convergence problems are detected, we assume simple random walks in our research in order to provide the most flexible framework for state variables. Thus, our first trial will be $r=1$ and the $\mathbf{D(w)}$ matrix becomes the identity matrix.

The relationship between the state variables and the measurements involves *time-varying model parameters* (congestion–dependent, since they are updated from sample travel times provided by equipped passengers). This is applied using a linear transformation that considers:

- The number of equipped passengers first detected in an equipped transit-stop *r* (linked/related to one or more origin zones *i*) during time intervals in *k ,..., k-M*, $q_r(k)$.
- *H<M time*-varying *model parameters* in the form of *fraction matrices* $\left[u_{ije,rs}^{h}(k)\right]$.

- The H adaptive fractions that approximate the travel time distribution between the pair of ICT sensors *(r,s)* notated as u_{rs}^{h} and that extend to $u_{ije,rs}^{h}$. These are updated from travel time measures provided by ICT sensors.

At time interval *k*, the values of the observations are determined by those of the state variables at time intervals *k, k-1, …, k-M*:

$$\mathbf{\Delta z}(k) = \mathbf{F}(k)\Delta\mathbf{g}(k) + \mathbf{v}(k), \tag{2}$$

where $\mathbf{v}(k)$ is white Gaussian noise with covariance matrix $\mathbf{R_k}$. $\mathbf{F}(k)$ maps the state vector $\Delta\mathbf{g}(k)$ onto the current blocks of measurements at time interval *k*: counts of equipped passengers between a pair of ICT sensors, accounting for time lags and congestion effects and balances for origin zone preservation flow, if available (for example, one origin zone identified as the only source of passengers for a transit stop). The solution should provide estimations of the OD passenger matrices between OD pairs for each time interval up to the *k*-th interval once observations of ICT equipped passengers at the bus-stops equipped with wifi antennas up to the *k*-th interval are available.

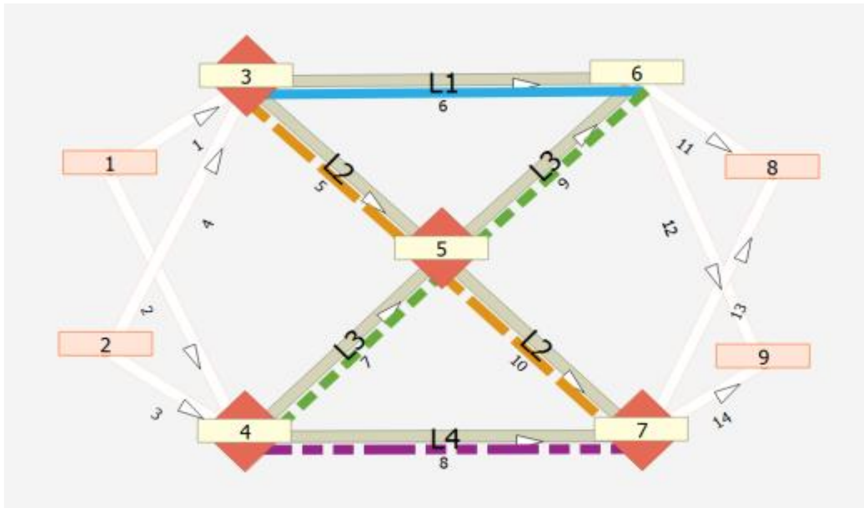
KF prediction of OD trips for ICT-equipped passengers up to some intervals ahead has to be considered and expanded according to historic profiles (for day-type and time-period), in order to feed a dynamic transit assignment tool that will provide the forecasted transit line loads, boardings/alightings at transit-stops in the short-term future. Here, we consider a 30 min forecasting horizon.

Description of the Test Network

The formulation to be detailed has been programmed as a MatLab prototype (named KFX3T). Proper coding has been verified with a toy network presented in Fig. **1**.

The OD matrix consists of four non-zero OD transit flows (1,8), (1,9), (2,8) and (2,9) (identified as 1 to 4). ICT sensors are assumed to be available at nodes 3, 4, 5 and 7 (sensor IDs are, respectively, 1 to 4). Transit lines are L1 to L4: L1 from 3 to 6, L2 from 3 to 7, L3 from 4 to 6 and L4 from 4 to 7,. Headway for lines L1 and L4 is 15 min and otherwise set to 10 min. Flow is distributed at origins using a *logit* model with scale parameter 0.2. Travel speed for all lines is 20km/h and boarding time is neglected to simplify the description of the example. Half headway for each line is incorporated into experienced travel times (travel impedance). In-vehicle travel times result in 10.8 min for segments on L1 and L4 and 7.5 min for each L2 and L3 segments.

The set of OD paths that compose, according to EMME (2013), the optimal transit assignment strategies for the whole period of study are described in Table 2. Thus, the number of OD pairs is 4, the number of OD paths is 10 and there are 5 feasible pairs of equipped transit stop captures *(r,s)*. We assume a subinterval of Δt=5 min for a 1 hour horizon of study.



OD	8	9
1	180	120
2	60	180

Fig. 1. Test network: link and node identifiers, ICT sensors (diamond nodes) and transit lines (L1 to L4)

Table 2. Description of OD paths related to optimal strategies in transit assignment

OD Path/ Optimal Estrategy	OD path lines	ODPath id in OD pair	OD pair	ICT id	OD Path/ Optimal Estrategy	OD path lines	ODPath id in OD pair	OD pair	ICT id
1/1	L1	1	1=(1,8)	1	**6/3**	L1	1	3=(2,8)	1
2/1	L3	2	1=(1,8)	2,3	**7/3**	L3	2	3=(2,8)	2,3
3/1	L4	3	1=(1,8)	2,4	**8/3**	L4	3	3=(2,8)	2,4
4/2	L2	1	2=(1,9)	1,3,4	**9/4**	L2	1	4=(2,9)	1,3,4
5/2	L4	2	2=(1,9)	2,4	**10/4**	L4	2	4=(2,9)	2,4

Table 3. Time-varying model parameters $u_{rs}^{h}(k)$: proportion of passengers arriving at stop *s* at time *k* from stop *r* in h time intervals

Orig in		***TT(r,s)(min) for k=***						**(ODpath id, ICT stop)**	$h\ s.t.\ u_{rs}^{h}(k)=1$ $\Delta t=5'$	$h\ s.t.\ u_{rs}^{h}(k)>0$ $\Delta t=5'$	$h\ s.t.\ u_{rs}^{h}(k)>0$ $\Delta t=5'$
i	***(r,s)***	*0*	*1*	*2*	*3*	*4*	...	*Any k*	*k=0,1,2*	*k=4*	*k>>4*
1	*[1,3]*	7.5	7.5	7.5	12.5	12.5	12.5	(4,3) (9,3)	1	1, 2	2
1	*[1,4]*	15	15	15	15	20	20	(4,4) (9,4)	2	2,3	3
2	*[2,3]*	7.5	7.5	7.5	12.5	12.5	12.5	(2,3) (7,3)	1	1, 2	2
2	*[2,4]*	10.8	10.8	10.8	10.8	18.3	18.3	(3,4)(8,4) (5,4)(10,4)	2	2,3	3
-	*[3,4]*	7.5	7.5	7.5	7.5	7.5	7.5	(4,4) (9,4)	1	$1\ \left(u_{34}^{1}(k)=1\right)$	$1\ \left(u_{34}^{1}(k)=1\right)$

Model Building for the Test Network

For each *[r,s]* pair of ICT sensors, $u_{rs}^{h}(k)$ account for temporal dispersion of experienced travel times, and they are applied to all OD paths linking equipped transit stops *(r,s)*. We also assume that the extended vector of states is the current one and that M, the number of Δt subintervals, is set to M=5. Initially, deviates of the OD path flows are set equal to the historic OD path flows, and 100% of equipped passengers is assumed for validation purposes; thus $\Delta g_{ije}(0)=0$. Time-varying model parameters $u_{rs}^{h_0}(0)$ at initialization are set according to segment line speeds (20km/h), and for *k*>0 they are defined as the average experienced travel impedances as indicated in Table 3. Then, if $u_{rs}^{h_0}(0)=1$ for h_0 equal to 1, then that would mean the *a priori* travel time to get to stop *s* from *r* for all the passengers is one time subinterval, in the range (Δt,2Δt] minutes, or in other words between 5 and 10 min.

Let $\Delta\mathbf{g(k)}$ be a column containing state variables for intervals $k, k-1, \ldots, k-M$ of dimension *(M+1)x10=(5+1)x10=60*. The extended vector state has 60 components in the example and, for *r* =1, the state variable equations are defined as a random walk which can be expressed for the extended state vector as a shifting operator (Eq. 1) by setting **D** as a 60x60 matrix with non-null submatrix components $\mathbf{I}_{10}$ (identity matrix 10x10).

$$\mathbf{D}=\begin{pmatrix}\mathbf{I}_{10} & 0 & \ldots & 0\\ \mathbf{I}_{10} & 0 & 0 & 0\\ 0 & \ddots & 0 & 0\\ 0 & 0 & \mathbf{I}_{10} & 0\end{pmatrix}$$

The time-varying linear operator relates state variables and current observations for time interval *k* in Equation (2). In the example, $\Delta \mathrm{z(k)}$ is a vector of dimension (*Q*+*L*)=5+1=6, since ICT sensors 1 and 2 (located at nodes 3 and 4) do not clearly identify each one of them as the natural receptor of either origin centroid 1 or 2. However, by adding up their captures, the total flow generated by origin zones 1 and 2 provides 1 conservation flow equation. We refine the details of Eq. (2):

$$\Delta\mathbf{z}(k) = \begin{pmatrix}\mathbf{AU(k)}^{\mathbf{T}}\\ \mathbf{E(k)}\end{pmatrix}\Delta\mathbf{g(k)}+\begin{pmatrix}\mathrm{v}_1(\mathrm{k})\\ \mathrm{v}_2(\mathrm{k})\end{pmatrix}=\mathbf{F}(k)\Delta\mathbf{g}(k)+\mathbf{v}(k) \qquad (3)$$

A discretization of the travel-time distributions between pairs of equipped transit stops *[r,s]* (*5 pairs*) in H=2 bins is enough in this case (the first bin h_0 is represented by *a priori* travel times). Then, if C_i is the number of OD paths originating in the network at entry centroid *i*, for $i=1,\ldots,I$, then C_1=5 and C_2=5 in our case, leading to the definition of one balance equation, through **B** and **E(k)** in Eq. (3) as:

$$\mathbf{B} = \left(\overbrace{11111}^{C_1=5} \quad \overbrace{11111}^{C_2=5} \right), \quad \mathbf{E} = [\mathbf{B} \quad \mathbf{0}_{10} \quad \mathbf{0}_{10} \quad \mathbf{0}_{10} \quad \mathbf{0}_{10} \quad \mathbf{0}_{10}]$$

A matrix in Equation (3) is composed by appending identity matrices of dimension 5, M+1=6 times:

$$\mathbf{A} = \left(\underbrace{\mathbf{I_Q} \quad \dots \quad \mathbf{I_Q}}_{M+1} \right)$$

and each $U(k-h)$ for $h=0,\dots,M$ models network structure and travel time delays in terms of fractions on travel time bins (time-varying model parameters affecting state variables whose OD subpaths are intercepted by some pairs of ICT-equipped transit stops *(r,s)* at *s* in interval *k*). Implementation guarantees that implicit structural restrictions are satisfied (see Equation 4):

$$\begin{array}{llll} u_{rs}^h(k) \geq 0 & \forall (r,s), h = 1 \dots H & u_{ije,rs}^h(k) \geq 0 & \forall (r,s), h = 1 \dots H \\ \sum_{h=1}^{H} u_{rs}^h(k) = 1 & \forall (r,s), \quad \forall k & \rightarrow \sum_{h=1}^{H} u_{ije,rs}^h(k) = 1 & \forall (r,s) \text{ captured path } ije \quad \forall k \end{array} \tag{4}$$

$$U(k-h) = \left(\underbrace{\begin{matrix} u_{ije,rs}^h(k-h) & \dots \\ \vdots & \ddots \end{matrix}}_{Q} \right) \quad \text{and} \quad \mathbf{U(k)} = \begin{pmatrix} U(k) & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & U(k-M) \end{pmatrix}$$

$\mathbf{U(k)}$ is (1+M)10 x (1+M)Q=6*10x6*5=60x30. In the example $u_{ije,rs}^h(k-h)>0$ is the fraction of the equipped passengers that were detected at equipped transit stop *r* (*h* intervals before *k*) and who arrive at sensor *s* at *k* interval (this fraction applies to all captured OD paths). For *k=1*, we define submatrices

$$U(1) = U(0) = U(-1) = U(-2) = U(-3) = U(-4) = \mathbf{0_{10x5}}$$

in $\mathbf{U(1)}$, since passengers that enter the system during subinterval *k=1* cannot reach any equipped transit stop *s* from any detector at an equipped stop *r (minimum time is 7.5 min)*. We skip a transit by applying *k=2,3*, but once *k=4* then all pairs *(r,s)* have already received experienced travel times from passengers, and approximations to travel time distributions are updated. For *k=4*, we have to average travel times in Table 3:

$$U(k-1)=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.3\widehat{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0.3\widehat{3} & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.3\widehat{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0.3\widehat{3} & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} U(k-2)=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.6\widehat{6} & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 \\ 0.6\widehat{6} & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.6\widehat{6} & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 \\ 0.6\widehat{6} & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 \end{pmatrix} U(k-3)=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 \end{pmatrix}$$

$$U(k)=U(k-4)=U(k-5)=\mathbf{0}_{\mathbf{10x5}}\ .$$

And for k>>M, we would have (always referring to Table 3 travel times):

$$U(k-1)=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} U(k-2)=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} U(k-3)=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$U(k)=U(k-4)=U(k-5)=\mathbf{0}_{\mathbf{10x5}}$$

And the right-hand side of Eq. (3) is fully described for the example. The left-hand side refers to deviates of counts for each pair *(r,s)* of ICT equipped transit stops (5 in the example), and for 1 conservation flow equation (last row) for all OD paths entering the system at interval *k* in Eq. (3). This concludes the model building details for the proposed formulation.

The standard report files related to OD flows, nodes, links, etc. in the EMME (2013) model were fitted to worksheet formats (fixed column *.csv* files) directly or by using python scripting. Fixed column *.csv* data files are read by KFX3T model building procedures. The KFX3T internal data model is split into MAT files, which are loaded as needed into the program as: Global.mat, Tuning.mat, Graph.mat, Demand.mat, Measures.mat. Additionally, two extra MAT files have been included for internal use in order to simplify the access to some critical structures: AccDem.mat (OD pairs and OD paths) and AccMes.mat (OD paths captured by each defined sensor and pairs of ICT sensors).

Conclusions

Model building for a KF formulation for estimating dynamic transit matrices, as proposed by the authors, has been detailed in a toy example. An EMME (2013) model for the test network was developed to feed the internal data model for the KFX3T prototype implemented in MATLAB. We developed a discrete event simulator to emulate passenger counts and travel times in the test network. KFX3T has been validated and convergence holds, but a fine tuning is needed. Sensibility to *a priori* OD flows per interval and co-variance matrices for state variables and counts are being tested. Testing on medium-sized networks is the next step to be undertaken in the immediate future.

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