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Heuristics for single machine scheduling under competition to minimize total weighted completion time and makespan objectives

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Abstract. This paper considers a single machine scheduling problem, where two agents compete for the use of a single processing resource. Each of the agents needs to process a set of jobs with the common resource to optimize their own objective function which depends on the completion time of its own jobs. The goal is to minimize the total weighted completion time of first agent subject to an upper bound on the makespan of the second agent. The problem is binary NP-hard. We propose three simple heuristic for the problem. These heuristics are based on shortest processing time rule, highest weight first rule and weighted shortest process time first rule. Numerical experiment is performed on randomly generated problem instances. Heuristic performances are evaluated by comparing it with the exact solution.

Keywords: scheduling; competing agents; heuristic; combinatorial optimization

Introduction

Scheduling is concerned with the allocation of limited resources. We study a class of scheduling problem with two agents competing for the use of common resources. The problem is encountered in many real world applications such as preventative maintenance

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(Leung *et al*., 2010), research and development (Baker and Smith, 2003), rescheduling (Hall and Potts, 2004) etc. We study a single machine problem in which two agents A and B compete for the single machine to get processed. Each agent has a set of jobs J_A and J_B to be processed by a single machine. Job set $J^A = \{J_1^A, J_2^A, \ldots J_{n_A}^A\}$ consist of n^A jobs and set $J^B = \{J_1^B, J_2^B, \ldots J_{n_B}^B\}$ consist of n^B jobs. The processing time of job J_j^A and J_j^B is denoted by p_j^A, $j = 1\ldots, n_A$, p_j^B, $j = 1\ldots, n_B$ respectively. Let the total process time of agents A and B is denoted by

$$P_A = \sum_{i=1}^{n_A} p_i^A$$

and

$$P_B = \sum_{i=1}^{n_B} p_i^B$$

We define the w_j^A to be the weight of job *j* and δ_j^A to be the density of job *j* where

$$\delta_j^A = {w_j^A}\big/{p_j^A}$$

We assume that the job in set J^A is numbered according to the density; hence, δ_1^A and $\delta_{n_A}^A$ is the highest and lowest density respectively.

For a given job sequence σ, lets define the completion time of job J_j^A and J_j^B by C_j^A and C_j^B respectively. The objective of agent A and B is denoted by $f^A(\sigma)$ and $f^B(\sigma)$ respectively. The objective function of the agent A is to minimize the total weighted completion time

$$f^A(\sigma) = \sum w_i^A C_i^A(\sigma)$$

and the objective function of agent B is to minimize the maximum completion time

$$f^B(\sigma) = C_{\max}^B = \max_{j \in J^B}\{C_j^B(\sigma)\}$$

In this paper, we minimize the total weighted completion time $f^A(\sigma)$ of agent A subject to an upper bound *Q* on the maximum completion time $f^B(\sigma)$ of agent B. According to the Agnetis *et al*. (2004) notations, the problem considered in this paper is denoted as a

$$1|C_{\max}^B \leq \text{Q}|\sum w_i^A C_i^A$$

Agnetis *et al*. (2004) proved that the problem is binary NP-hard. The purpose of this paper is to explore different heuristics for the problem. The problem is although considered NP-hard but it can be solved in pseudo polynomial time. This paper investigates that how easy it is to solve the problem.

The remainder of the paper is organized as follows: We review the related literature in section 2. In section 3, we propose three heuristics to solve the problem under study. In section 4, we perform numerical analysis and compare the performance of different heuristics followed by conclusions in section 5.

Literature Review

The problem of two competing agents was introduced by Baker and Smith (2003) and Agnetis *et al*. (2004). These papers considered different objective functions for each agent. The initial papers provided polynomial time algorithm and NP-hardness proof for different problems.

There are number of papers published after these papers considering different aspect of scheduling problem such as release date related objective function (Ng *et al.* (2006), Leung *et al*. (2010), Yin *et al*. (2012b) and Yin *et al*. (2013)); batch scheduling (Mor and Mosheiov (2011) and Li and Yuan (2012)); earliness and tardiness related objective function (Mor and Mosheiov (2010) and Gerstl and Mosheiov (2013)); varying process time (Liu *et al*. (2010a), Liu *et al*. (2010b), Wan *et al*. (2010), Gawiejnowicz *et al*. (2011), Yin *et al*. (2012a)); learning effect on process time (Cheng *et al*. (2011), Li and Hsu (2012)); flowshop and parallel machine problem (Wan *et al*. (2010), Leung *et al*. (2010), Lee *et al*. (2011), Mor and Mosheiov (2014)). We study the problem with total weighted completion time and makespan objectives. This problem is studied by Agnetics *et al.* (2009). They developed branch and bound based lagrangian approach to solve the problem. They also provided the recursive equation for dynamic programming approach which can solve the problem in pseudo polynomial time. In our knowledge there is no heuristic solution proposed for this problem. We propose simple heuristic solution (inspired from the property of single agent problem) to solve this problem.

The proposed heuristic algorithm

In this section we present three heuristics based on 1) shortest process time first rule, 2) highest weight first rule and 3) weighted shortest process time first rule. We first present two property of the problem structure from previous studies (Baker and Smith (2003) and Agnetics *et al*. (2004)) followed by a lemma for improving the solution quality.

Property 1: In an optimal schedule σ^* , all the B jobs are scheduled consecutively.

Property 2: In an optimal schedule σ^* , jobs in J^A_{prec} and jobs in J^A_{succ} are ordered by no-increasing values of the ratio of

$$\delta_i^A = w_i^A / p_i^A .$$

As a consequence of property 1, the *B*-jobs can be treated as a single job J_B with process time of P_B and thus the structure of an optimal solution to the problem is denoted by $\{J^A_{prec}\}\{J_B\}\{J^A_{succ}\}$. Here, J^A_{prec} represents the set of *A*-jobs proceeding *B*-jobs and J^A_{succ} represents the set of *A*-jobs succeeding B jobs. These properties provide a background for our proposed heuristics. In the proposed heuristics, we first divide the *A*-jobs in two sets J^A_{prec} and J^A_{succ}. These jobs are arranged in non-increasing order of ratio of δ_i^A. Then the sequence is represented by $\{J^A_{prec}\}\{J_B\}\{J^A_{succ}\}$. We also use following lemma to improve the solution quality.

Lemma 1: In a given schedule σ, if a job from J^A_{succ} can be transferred to J^A_{prec} feasibly (without violating the upper bound Q for the completion time of B-jobs) then the resultant schedule improves the solution.

This lemma can be proved by simple job interchange scheme. Now we present three heuristics used in this paper to solve the problem under investigation.

Shortest process time first heuristic

This heuristic is based on notion that the job with the shortest process time should be scheduled first to minimize the total weighted completion time objective. In this heuristic the jobs from agent A are first arranged in increasing order of process time p^A_j. Then the first k jobs with total process time less than $Q - P_B$ is included in set J^A_{prec} and remaining jobs are included in J^A_{succ}. The jobs in sets J^A_{prec} and J^A_{succ} are arranged in non-increasing order of ratio δ^A_i. The resultant sequence is represented by $\{J^A_{prec}\}\{J_B\}\{J^A_{succ}\}$. Finally lemma 1 is used to improve the solution quality.

Highest weight first heuristic

This heuristic is based on the notion that job with higher weight is scheduled first to minimize the total weight completion time objective. In this heuristic the jobs from agent A are first arranged in decreasing order of weight w^A_j. The first k jobs with total process time less than $Q - P_B$ is included in set J^A_{prec} and remaining jobs are included in J^A_{succ}. The jobs in sets J^A_{prec} and J^A_{succ} are arranged in non-increasing order of δ^A_i. The resultant sequence is represented by $\{J^A_{prec}\}\{J_B\}\{J^A_{succ}\}$. Finally lemma 1 is used to improve the solution quality.

Weighted shortest process time first heuristic

This heuristic uses the well-known weighted shortest processing time first (WSPT) rule of single agent scheduling problem. In this heuristic, A-jobs are first arranged in increasing order of δ^A_i. Then the first k jobs with total process time less than $Q - P_B$ is included in set J^A_{prec} and remaining jobs are included in J^A_{succ}. The resultant sequence is represented by $\{J^A_{prec}\}\{J_B\}\{J^A_{succ}\}$. Note that the jobs in sets J^A_{prec} and J^A_{succ} are already arranged in non-increasing order of δ^A_i. Finally lemma 1 is used to improve the solution quality.

Numerical Analysis:

The proposed heuristic is tested on randomly generated 10 problem instances in which number of jobs for agent A and agent B varies from 10 to 100. All the parameters are generated in the way they were generated by Agnetis *et al*. (2009). All process times and weights are considered integer and generated uniformly in [1, 25]. The value of Q is generated by setting $+Q=\alpha(\mathrm{P}^A+\mathrm{P}^B)+\mathrm{P}^B/2$, where α is uniformly generated between [0.4, 0.6].

The proposed algorithm were coded in C++ and implemented on AMD Opteron 2.3 GHz with 16 GB of RAM. The performances of these heuristics are evaluated by comparing it with optimal solution obtained using dynamic programming based algorithm mentioned by Agnetics *et al*. (2009). We used relative percentage deviation (RPD) (representing the deviation of the heuristic solution from the optimal solution) to evaluate the performance of heuristics. We use following notations for result reporting.

OPT: Optimal solution for the problem
HS1: Shortest processing time based heuristic
HS2: Highest Weight based heuristic
HS3: Weighted shortest process time based heuristic
ABS: Absolute value of the solution obtained by the heuristics
RPD: Relative percentage deviation

The results are reported in Table 1. We do not include CPU time because all the problem instances are solved in fraction of seconds.

Table 1. Computational results for the problem

n^A	*Q*	*OPT*	*HS1*		*HS2*		*HS3*	
			ABS	*RPD*	*ABS*	*RPD*	*ABS*	*RPD*
10	220	4456	7455	67.30	4456	0.00	4456	0.00
20	339	52190	52190	0.00	52245	0.11	52190	0.00
30	601	112581	151481	34.55	118444	5.21	113968	1.23
40	874	211420	247064	16.86	223208	5.58	212495	0.51
50	942	294877	336494	14.11	313008	6.15	294877	0.00
60	1136	456688	522578	14.43	461921	1.15	457803	0.24
70	1546	433817	531482	22.51	440128	1.45	433817	0.00
80	1876	508544	659747	29.73	531880	4.59	510702	0.42
90	2039	608508	773111	27.05	623124	2.40	613200	0.77
100	2230	561338	814484	45.10	564049	0.48	561338	0.00
Average			409608	27.16	333246.30	2.71	325484.60	0.32

We use average value of relative percentage deviation to compare different heuristics. The results reported in Table 1 show that the HS1, HS2 and HS3 are away from the optimal solution by 27.16 %, 2.71 % and 0.32 % respectively. Among three heuristic presented in this paper, HS3 heuristic (based on WSPT rule) performs better than other heuristics. The performance of shortest process time based heuristics (i.e., HS1) is poor with 27 % RPD value. While the performance of highest weight time based heuristic (i.e., HS2) is much better than HS1 but inferior to the HS3. The objective function of weighted process time requires that the jobs with shortest time and higher weight are processed first. The heuristic HS1 and HS2 exploit these properties but not fully while HS3 heuristic fully exploits the property of weighted process time objective function. Therefore HS3 performs better than HS1 and HS2. The results reported in Table 1 further shows that the average RPD for HS3 is just 0.32 %. These results indicate that HS3 is able to produce the solution close to the optimal solution for the problem. The improvement scheme proposed by lemma 1 also plays a role on heuristic solution. Before introducing the improvement scheme in HS3 the average RPD was 1.41 %, however, after introducing the improvement scheme the average RPD reduces to 0.32%. The small value of RPD for HS3 also indicates that the problem under consideration is although NP-hard in theory but in practice it is not very hard to solve.

Conclusions

We studied single machine scheduling problem with two competing agents with objective of minimizing the total weighted completion time of first agent subject to an upper bound on the makespan of the second agent. We proposed three heuristics based on shortest process time first rule, highest weight time first rule and the weighted shortest process time first rule. The heuristic solutions are evaluated using randomly generated problem instances. The computational results show that the heuristic based on weighted shortest process time rule performs better than other heuristics. The heuristics based on weighted shortest process time rule is able to produce solution close to the optimal solution. The results of heuristic solution indicate that the two agent problem with weighted completion time objective is although NP-hard in theory but in practice the problem is easy to solve.

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