

Hybrid-demand queueing for commuter parking

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Abstract

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Registered (finite-source) commuters and (infinite-source) visitors share a commuter parking lot. We develop a novel, hybrid-demand equilibrium queueing model, with demand dynamics, for use in traffic-engineering these lots. Our transient version can help analyze the daily lot startup process. Finally, we propose criteria and take initial steps to evaluate our models. While the problem is not new, there are essentially no other published analytical results or detailed studies regarding commuter lot engineering.

Introduction

With Monmouth University (MU) as our study partner, we previously developed an enterprise-wide model and prototype software system for optimized academic scheduling, with explicit consideration of the school's shared commuter/visitor parking lot. Pack *et al* (2011) reported that our optimization freed up more than **20%** of MU's classrooms for other uses, even though most MU administrators had felt that space was "tight". However, MU emphasized that such reuse opportunities, potentially worth millions of dollars in revenues, could not be realized unless we also helped increase throughput in the *existing* commuter parking lots, which already had a high *average* utilization of about **80%**. The result: by reducing the *average* time students must spend on campus and flattening program schedules, our optimization increased by **25-35%** potential MU lot-flow rates. Perhaps surprisingly, that paper is not only the first publication on commuter parking lot management for an academic institution but, apparently, one of very few papers on commuter parking engineering itself. In this paper, motivated by analyses of sampled MU parking data, we propose and illustrate a hybrid-demand *equilibrium* queueing model, with demand dynamics reflecting daily and weekly traffic variations, that can be used to *traffic-engineer* commuter lots. Because most commuter parking systems fill up to near capacity each day, we also develop a *transient* version for better understanding the daily startup process. Finally, we *begin* evaluating our models via the following criteria:

- **Modeling Accuracy:** Do the models capture/reproduce the essential elements/ attributes of the commuter-parking problem? Given parameter values, do the models produce meaningful results in the region of engineering interest, i.e., is the typical/average "*modeling error*" small? In essence, this criterion looks at systematic, and not stochastic, modeling errors.
- **Engineering Accuracy:** Considering various sources of uncertainty, do the model's results match reality? That is, performance assessment and capacity-engineering decisions should account for the effects (usually *reduced capacity*) of: systematic modeling error that may bias results; demand dynamics (a stochastic part of our model); and measurement, forecast and parameter-estimation errors.
- **Model Validation:** Ultimately, do the model and associated methods work, i.e., provide sufficient value in the parking lot management process? This criterion includes the effects of the other criteria.

In this paper, we focus primarily on Modeling Accuracy, whereas Pack (2015) will use existing data from MU parking lots to take a first look at Engineering Accuracy. More research, including simulations and additional field studies, will be required to fully evaluate, refine and implement our models.

Parking lot design is often cited without much detail, as a good application of queueing theory. The most common (equilibrium) analytical model is the "Erlang B" (M/M/N/N) with *infinite-source* Poisson (Markovian) arrivals, exponential (Markovian) parking times and N parking spaces with no waiting or overflow area. These assumptions make analysis and software implementations fairly straightforward. However, in practice, arrivals may actually be *finite-source* (especially for *commuters*), "peaked" (not random), and dependent on the time and day of week; and waiting or overflow areas may be available. Arnott *et al* (1991) has an analytical *deterministic* commuter-parking model, with a focus on pricing. Because, commuter-parking systems, especially those on an academic campus, may have many lots with complex parking strategies, researchers often use detailed simulations; see, e.g., Harris *et al* (1997). Boxma (1986) assumes a *hybrid-demand*

(non-parking) system; however, his queueing model differs from ours in that he has a *single* server and *infinite* waiting space. Finally, none of these papers addresses the effects of demand, measurement and model uncertainty on the quality of traffic engineering results.

Shared Commuter Parking: Our Definition

Registered *commuters* regularly try to park in their shared **Lot L**, which can hold N cars. When space is available, they stay for many hours, leave and return in a day or so to repeat the process. *Visitors* show up more randomly; when space is available, they stay for several hours and do not return. When space is not available, a *commuter or visitor* will may seek space in virtual overflow **Lot O**, with N_o spaces; whether successful or not in finding space in **O**, *commuters* will, next, return home until the following day, while *visitors* leave the system. Owners may have many goals for their parking lots, including large profits, low costs, business efficiency and employee/community support. However, undoubtedly, one goal is to have high lot (investment) utilization, while also providing excellent service to registered users. Unfortunately, with limited access to good tools, parking lot managers may limit the number of registered users to be close to lot capacity, N , while also severely restricting visitors, *unnecessarily* reducing average lot utilization.

Hybrid-Demand Equilibrium Queueing Model for Commuter Parking

We develop a new *equilibrium* hybrid-demand queueing model, with demand dynamics, that is consistent with most elements of our above definition and which could provide better support for the joint goal of good service and high lot utilization. Our *equilibrium* model includes finite-source (commuters = Type 1) and infinite-source (visitors = Type 2) demands that share a lot's parking capacity. We begin with a *static* version with fixed parameters and then present a *dynamic* version (variable parameters) along with the elements of a typical traffic engineering process that must reflect the impact of various uncertainties.

In our *static* version, we assume that each commuter parking **Lot L** operates independently with N_L spaces available for both visitors and s_L registered users. (We usually omit the subscripts **L**.) Let \mathbf{J}_1 and \mathbf{J}_2 be the random number of Type 1 and Type 2 cars in **Lot L** at some point in time, assuming that cars not finding a space are "overflowed" from (leave) **L**; all Type 1 customers may request access to **L** a *random* time later (e.g., tomorrow) after service or overflow. As Cooper (1981, page 131) motivates, there exists an equilibrium joint "product form" distribution at a random point in time:

$$P_{s,N,a_1,a_2}(j_1, j_2) = P[J_1 = j_1, J_2 = j_2] = \binom{s}{j_1} a_1^{j_1} \frac{a_2^{j_2}}{j_2!} C_{s,N,a_1,a_2} \text{ for } 0 \leq j_1 \leq \min(s, N), 0 \leq j_2 \leq N - j_1 \quad (1)$$

and

$$C_{s,N,a_1,a_2}^{-1} = \sum_{k=0}^{\min(s,N)} \binom{s}{k} a_1^k \sum_{r=0}^{N-k} \frac{a_2^r}{r!}.$$

Type 1 "*quasi random*" *idle source load* is $a_1 = \gamma/\mu_1$, γ is the independent arrival rate for each *idle* source and $1/\mu_1$ is the average park time; and Type 2 offered load is $a_2 = \lambda/\mu_2$, λ is Poisson demand arrival rate and $1/\mu_2$ is the average park time. While, usually, service times are assumed to be exponential, this is not critical to most analyses. This result can be readily proven from the system birth-death (B-D) equations and boundary conditions, assuming that "births" (arrivals) occur when a Type-1 or Type-2 source requests a parking space and one is available. Finite-source arrivals occur only when there is an idle Type 1 source. Deaths correspond to service completions; overflows occur when arrivals do not find a parking space. Then, analogous to other hybrid-demand systems, respective overflow probabilities are:

$$\begin{aligned} \Pi_{s,N,a_1,a_2}^{(2)} &= P[\text{arriving Type 2 car finds no space}] = P[J_1 + J_2 = N \mid \text{Type 2 arrival}] \\ &= \sum_{j_1=0}^{\min(s,N)} P_{s,N,a_1,a_2}(j_1, N - j_1) \text{ for } s \geq 0 \end{aligned} \quad (2)$$

$$\Pi_{s,N,a_1,a_2}^{(1)} = \Pi_{s-1,N,a_1,a_2}^{(2)} = \sum_{j_1=0}^{\min(s-1,N)} P_{s-1,N,a_1,a_2}(j_1, N-j_1) \text{ for } s \geq 1. \quad (3)$$

Clearly, then, $\Pi_{s,N,a_1,a_2}^{(1)} \leq \Pi_{s,N,a_1,a_2}^{(2)}$. Next, we calculate L_{s,N,a_1,a_2} , the average number of cars in Lot **L**, as

$$L_{s,N,a_1,a_2} = L_{s,N,a_1,a_2}^{(1)} + L_{s,N,a_1,a_2}^{(2)} = \sum_{j_1=0}^{\min(s,N)} \sum_{j_2=0}^{N-j_1} (j_1 + j_2) P_{s,N,a_1,a_2}(j_1, j_2) \quad (4)$$

where the superscripts in (4) reference the respective terms for Type 1 and 2 demands. Then, the average *percent utilization* is

$$\rho_{s,N,a_1,a_2} = 100 L_{s,N,a_1,a_2} / N.$$

This “static” model seems to have *most* elements needed to characterize the performance of commuter parking lots with hybrid demands. However, as we discuss in our Evaluation section, below, we must be careful how we apply these results because our model omits **O** and allows spurious returning demands. We’ll see that Π , L and ρ can be erroneously large, but mostly for parameter values that may be inconsistent with typical commuter behaviors. To minimize this concern, we offer a more optimistic/ realistic (**O** exists) performance measure, the proportion of demand served by **O**:

$$PT_{s,N,\tilde{N}_O,a_1,a_2} \approx \frac{L_{s,N+\tilde{N}_O,a_1,a_2} - L_{s,N,a_1,a_2}}{L_{s,N+\tilde{N}_O,a_1,a_2}}, \quad (5)$$

where \tilde{N}_O , the number of spaces in Lot **O**, is large enough to capture essentially all of Lot **L**’s overflows (i.e., PT does not change significantly when we further increase \tilde{N}_O). Note that (5) would be correct for $\Pi^{(1)}$, $\Pi^{(2)}$ and average time congestion if both user groups were Poisson with no queue (i.e., Erlang B).

To better reflect the variation of demands, both within the day and across the days of the week, we now develop a *dynamic* version. We assume there may be more than one engineering period (e.g., daytime vs. evening) so that not all daily demands need be in one model. We treat the variations in commuter and visitor parking demands separately. First, not all registered users (e.g., students or workers) need access to **L** for all hours in a day or all days of the week, and few would need access more than once a day. In fact, for most commuters, parking requirements are strongly related to their course or work schedule. Thus, we assume that S , the number of **Type 1** sources, is a random variable in (1)-(5). Specifically, we assume that, *for each engineering period*, S has a density function, $f_S(s)$, where $0 \leq s \leq s_{max}$ and s_{max} is the number of cars registered to use **L**. Similarly, for the Type 2 users, A_2 is a random variable with density function, $g_{A_2}(a_2)$ for $a_2 \geq 0$. However, we assume a_1 is fixed because it is relatively stable, for an “average commuter,” in each engineering period. Since, f and g are distributions over “space”, we claim a form of ergodicity to let them represent *time* variability over the day/week. Then, assuming S and A_2 independent of each other:

$$\Pi_{s_{max},N}^{(1)} = E_{S,A_2} [\Pi_{S,N,a_1,A_2}^{(1)}] = \sum_{s=1}^{s_{max}} \int_{a_2=0}^{\infty} \frac{\Pi_{s,N,a_1,a_2}^{(1)} f_S(s)}{1 - f_S(0)} g_{A_2}(a_2) da_2 \quad (6)$$

$$\Pi_{s_{max},N}^{(2)} = E_{S,A_2} [\Pi_{S,N,a_1,A_2}^{(2)}] = \sum_{s=0}^{s_{max}} \int_{a_2=0}^{\infty} \Pi_{s,N,a_1,a_2}^{(2)} f_S(s) g_{A_2}(a_2) da_2 \quad (7)$$

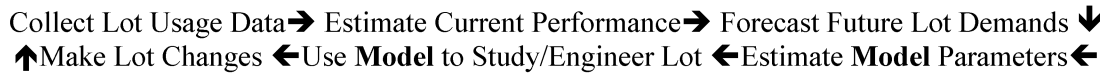
$$L_{s_{max},N}^{\#} = E_{S,A_2} [L_{S,N,a_1,A_2}] = \sum_{s=0}^{s_{max}} \int_{a_2=0}^{\infty} L_{s,N,a_1,a_2} f_S(s) g_{A_2}(a_2) da_2 \quad (8)$$

Average lot utilization is given by $\rho_{s_{max},N}^{\#} = 100 L_{s_{max},N}^{\#} / N$. Finally, analogous to (5),

$$PT_{s_{max},N,\tilde{N}_O}^{\#} \approx \frac{L_{s_{max},N+\tilde{N}_O}^{\#} - L_{s_{max},N}^{\#}}{L_{s_{max},N+\tilde{N}_O}^{\#}}, \quad (9)$$

where, as above, \tilde{N}_o is sufficient capacity for virtual overflow **Lot O**. Then, a design objective might be stated as $\Pi_{s_{max},N}^{(1)} \leq b^\#$ or $PT_{s_{max},N,N_0}^\# \leq b^\#$. As emphasized in the Introduction, the challenge for commuter lots is to achieve good service, $IT^{(1)}$ or $(PT^\#) \leq b^\# = .01$, and high utilization, $\rho^\# \approx 80\%+$, even for $s_{max} > N$.

Thus, the dynamic version reflects the very real phenomena of daily and weekly variations in commuter and visitor lot-use patterns for a given engineering period. As you might expect, for a given average design objective, $b^\#$, this variability effectively reduces lot capacity compared to that for the static model. To illustrate the point, if we use the assumptions, made in our Modeling Accuracy discussion below, we find that demand dynamics alone reduce lot capacity $\bar{A}_2^\#$ for visitors by a factor of 3 compared to a_2^* for a static model, i.e., $\bar{A}_2^\# = a_2^*/3$ at $b^\# = .01$. This impact would need to be taken into account in a traffic engineering process that might, for example, regularly repeat the following cycle for a given lot:



In Pack (2015), we discuss this process using sampled MU data to illustrate key ideas and initial methods. We also highlight the significant role that various uncertainties, e.g., demand dynamics and measurement, model and estimation errors, must play in an effective performance management and engineering process.

Hybrid-Demand Transient Queuing Model

The *equilibrium* conditions in the above model are in effect after the daily startup process. We now introduce a (simplified) *transient* model that could help administrators understand (and improve) startup performance, e.g.: how the demand mix affects lot fill rates; how long it takes for the lot to become congested; and best lot-space allocation strategies. The transient model ignores both capacity limits and possible service completions (“deaths”), i.e., we assume an infinite capacity birth-only, transient birth-death (B-D) model with *fixed* (static) parameters N, s (say, s_{max}), γ and λ . Given these assumptions, the respective demands at time $t, J_1(t)$ and $J_2(t)$, can be considered independent. Clearly, $J_2(t)$ is Poisson with mean λt . However, we found no citation for $J_1(t)$, which has a binomial form:

$$P_1(j, t) = P[J_1(t) = j] = \binom{s}{j} e^{-s\gamma} [e^\gamma - 1]^j \text{ for } 0 \leq j \leq s, \text{ and } 0 \text{ for } j > s. \tag{10}$$

Equation (10) satisfies the usual B-D condition:

$$\frac{dP_1(j, t)}{dt} = \gamma_{j-1} P_1(j-1, t) - \gamma_j P_1(j, t)$$

and boundary condition: $P_1(0,0)=1$, with $\gamma_j = (s-j)\gamma$ for $0 \leq j \leq s$ and 0 for $j > s$. We will want to analyze $Q_{s,\gamma,\lambda}(N,t) = P[J_1(t)+J_2(t) \geq N]$ as a measure of how the startup process approaches system congestion, given lot capacity N . We find that

$$Q_{s,\gamma,\lambda}(N,t) = \sum_{j=0}^{\min(s,N)} P_1(j,t) \sum_{i=N-j}^{\infty} P_2(i,t) + H(s-(N+1)) \sum_{k=N+1}^s P_1(k,t), \tag{11}$$

with step function $H(x)=1$ for $x \geq 0$ and 0 otherwise. We postpone until future publications, e.g., Pack (2015), detailed examples of the use of the transient queuing models. However, our initial studies, using fairly noisy data, suggest that it predicts fairly well the timing of MU’s congestion peaks and that, as you might expect, (11) seems much more sensitive to γ than to λ .

Evaluation of Models: Criteria and Initial Steps

This paper focuses on Modeling Accuracy for the *equilibrium* dynamic model. However, in order to provide insight into the difference between our static and dynamic versions, we include one aspect of Engineering Accuracy: how stochastic demand dynamics create uncertainty that reduces engineered lot capacity. The issues raised here for the equilibrium case mostly do not apply to our *transient* model; hence, we postpone its discussion until Pack (2105), which deals with Engineering Accuracy and data uncertainty for both equilibrium and transient versions. In addition, because Engineering Accuracy

depends on Modeling Accuracy, some of the examples in this paper are revisited in Pack (2015), where we increase N to 228, consistent with one of MU’s lots, and make direct use of actual MU lot measurements. Finally, the evaluation discussions in both papers necessarily illustrate how the models provide insight into engineering tradeoffs in lot design and management. Model Validation will still require considerable work in the future.

Modeling Accuracy focuses on systematic modeling errors that may bias results. Specifically, for the equilibrium model, we are mostly concerned with attributes of the commuter-parking problem that are, or are not, *explicitly* and *appropriately* incorporated in the model. Here, we cover five closely-related aspects of modeling accuracy: (i) overall consistency of model with our definition of commuter parking in a shared **Lot L**; (ii) (possible) omission of virtual overflow space, **Lot O**; (iii) spurious returning demands that are inconsistent with commuter behavior; (iv) biasing effects of large values of parameter, a_1 ; and (v) ability to predict the important condition of good service at high levels of utilization. We first look at issues (i) – (iv) together, and then (v). To illustrate our points, we use fairly small example systems ($N=100$), and guestimates, based on our experience with MU, of model parameter values. Let’s consider, “under what conditions, i.e., parameter values, network sizes and configurations, are the equilibrium models, (6)-(9), both appropriate and accurate enough for engineering purposes?” First, an assumption of *equilibrium* makes sense, especially for capacity engineering, if we ignore the lot’s short periods of start up (and shut down). This is commonly done at our trial site, MU, because lot data are not even collected for the first two (transient-phase) hours of the day. Second, we look at model/definition consistency, **L**’s overflow space, **O**, and spurious demands. When there is *no* overflow space (**O**), our model is largely consistent with the commuter definition because cars, completing service or overflowing, seek parking space again, in **L**, at a *random* time later; visitors, whether completing service or overflowing, exit the system. However, in reality (e.g., MU’s operation), overflowing commuter and visitor cars usually seek alternative parking in large virtual **Lot O**; whereas, the model would either ignore **O** or treat **O** and **L** together as a system. Whether **O** exists or not, by allowing commuter cars to return a *random* time later, the model erroneously (in most cases) allows multiple (*spurious*) parking attempts (“retries”) by a given user in **L** on the same day. We now motivate and use examples to show how these three *modeling errors* issues (**O**, spurious demands, large a_1) can be minimized by: using $PT^\#$ (i.e., (9)) for engineering since it assumes that **O** exists; keeping Type 1 demands, per source, small, say $a_1 \leq 3$; and engineering each **Lot L** to have few overflows, say, $b^\# = .01$, so that $IT^{(1)} \approx IT^{(2)} \approx PT^\#$. The motivation: The use of $PT^\#$ not only recognizes **O**; it also reduces spurious demands because only commuters *completing* service become active sources again; however, *some* may return the same day. It should be clear that, for commuters, surely $a_1 = \gamma/\mu_1 \leq 3$. This is because, for commuters, $\gamma \leq 1$ *attempt/ (engineering-period)* and $1/\mu_1$ is probably on the order of an *engineering-period*, where an engineering period might be something like 8am – 6pm. We now use examples and graphs to show how these strategies work together. For our dynamic model, we assume/guestimate that $b^\# = .01$, $N=100$, $\tilde{N}_o = 40$ and that $f_s(s)$ is a truncated negative binomial with $0 \leq s \leq s_{max} = 110$, the origin at s_{max} , and the open end directed towards $s=0$. (Note: we have found that f_s truncation effects can be ignored if $s_{max} > N \geq 100$.) We also let $z_s = var(S)/E(S) = 1$ and $\beta_s = E(S)/s_{max} = .85$, so that $E(S) \approx 94$.

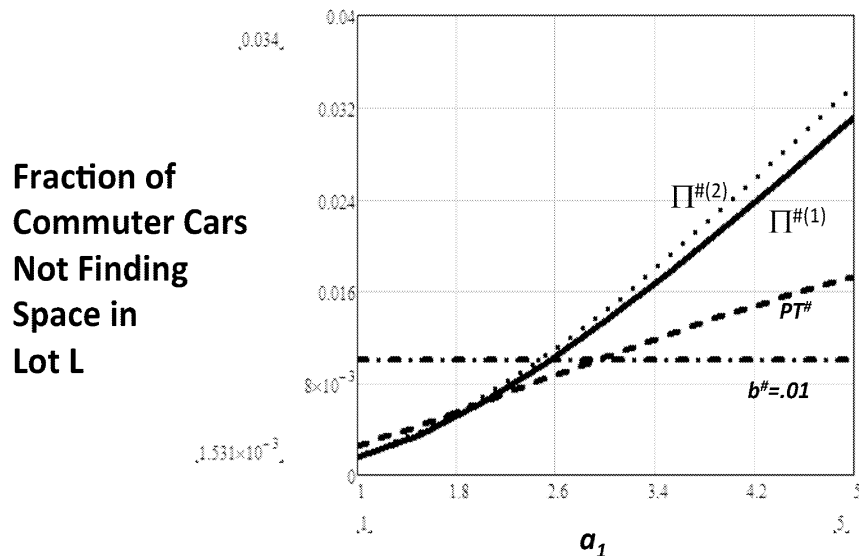


Fig.1. Various Probabilities vs. a_1

Fraction of
Commuter Cars
Not Finding
Space in
Lot L

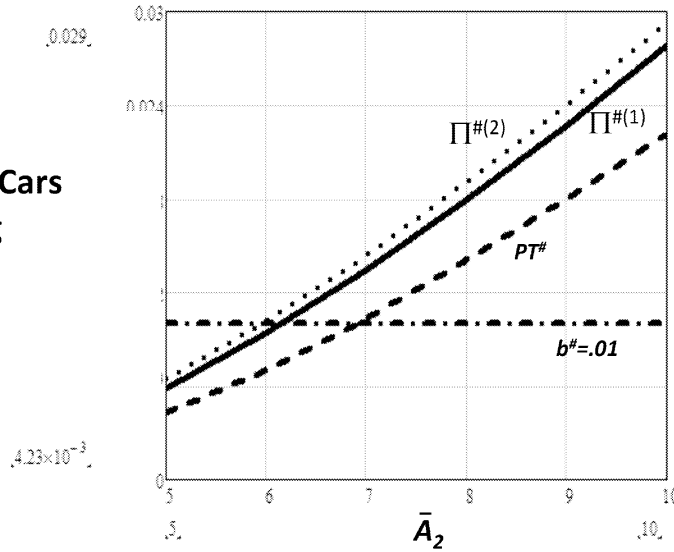


Fig.2. Various Probabilities vs. \bar{A}_2

In Figure 1, we plot $\Pi^{(1)}$, $\Pi^{(2)}$, $PT^\#$ and $b^\#=.01$ versus a_1 for $\bar{A}_2=7$, which is large enough to cause some congestion. We see that, for $a_1 \leq 3$, $\Pi^{(1)} \approx \Pi^{(2)} \approx PT^\#$, but that the Π 's and $PT^\#$ diverge for $a_1 \gg 3$. In Figure 2, we have a similar plot, except that we vary \bar{A}_2 and fix $a_1=3$. Again, curves $\Pi^{(1)}$, $\Pi^{(2)}$, $PT^\#$ are fairly close to each other, especially near $\bar{A}_2^\#$, the value at $b^\#=.01$. Using $\Pi^{(1)}$ suggests an $\bar{A}_2^\#$ capacity of 6.0, while $PT^\#$ is more optimistic with $\bar{A}_2^\# \approx 7.0$. These differences would be even smaller if $a_1 \ll 3$ or $b^\# \ll .01$. Clearly, we need much more work on Modeling Accuracy, including analysis, simulations and field studies. For example, while we have argued here that the effects on Modeling Accuracy of omitting **O** may be small, it might be helpful to extend our model to include **O** explicitly. We can write *four-variable* B-D equations that include finite overflow space, **O**, but it is probably quite difficult to obtain analytical results. Kosten has results for a similar system, assuming all demands are Poisson and $N_o = \infty$ (Cooper (1981, Chapter 4)).

Regarding issue (v)-- Can our model accurately represent the usual commuter parking system property: “good service (e.g., $b^\#=.01$) with high average facility utilization ($\rho^\# \approx 80\%$)” even when visitors use the lot ($\bar{A}_2 > 0$) and commuter registrations exceed lot capacity ($s_{max} > N$)? First, a system with only Poisson demands could *not* support this goal (service would deteriorate rapidly for high utilizations). Second, with hybrid demands, as various uncertainties rise, it *may be hard* to achieve high utilization unless, e.g., we greatly reduce visitor traffic limit $\bar{A}_2^\#$ at $b^\#=.01$. Thus, as an existence proof for *hybrid* demands, we have found that with assumptions similar to those in Figure 2, $P^\#$ (or $\Pi^{(1)}$) $\approx b^\#=.01$ and $\rho^\# \approx 75 - 80\%$ when $s_{max}^\# = 110$ and $\bar{A}_2^\# = 7$. We revisit this issue in Pack (2015) for systems more reflective of MU’s operation.

Engineering Accuracy addresses the effects of various *uncertainties* on engineering: systematic modeling error that may bias results; demand dynamics (a stochastic part of our model); and measurement, forecast and parameter estimation errors. These uncertainties impact critical engineering questions, e.g.: “is the lot meeting performance objectives?”; “do we need to add capacity/how much/where?”; and “are the risks of your decisions acceptable?” Clearly, Engineering Accuracy is affected directly by Modeling Accuracy. As we argued above, uncertainties due to *systematic* modeling errors can probably be minimized in parameter regions of engineering interest. So, we now consider how uncertainty due solely to demand dynamics affects engineered lot capacity. To do so, we created (do not show) a graph similar to Figure 2 for the static model ($a_1=3$, $s=E(S)=94$). We found the engineered limit for visitor traffic, $\alpha_2^\# \approx 20$, is 3 times $\bar{A}_2^\# = 7$ at $b^\#=.01$ in Figure 2. *Key insight*: system capacities, e.g., control $\bar{A}_2^\#$ (or $s_{max}^\#$) at $b^\#$, will be further reduced, beyond the $\bar{A}_2^\#$ factor of 3 for demand dynamics, if z_s increases, α_{A2} decreases or, as in Pack (2015), we account for measurement, model and estimate uncertainties. In particular, we must consider the effects of uncertainty, due to measurement/forecast errors, on model-parameter estimates and engineering results. This is quite relevant because the data, obtained from MU and utilized in our early analyses, were from *bi-hourly, manual* lot counts. Clearly, advances in data collection technologies, including intelligent cars, sensor networks and wireless, will help reduce uncertainties and improve commuter-lot engineering. **Model Validation** will require much work after the other two evaluation criteria are better understood.

Summary and Future Work

There are several major contributions of this paper. First, we propose and illustrate a novel hybrid-demand *equilibrium* queueing model, with demand dynamics, that can be used for commuter parking lot traffic engineering. We also present a simplified transient queueing model that can provide insights into startup process performance. Then, we took a first look at evaluating our models in categories: *Modeling Accuracy*, *Engineering Accuracy* and *Model Validation*, with focus here on issues and examples for the first criterion. We also looked at one aspect of Engineering Accuracy: how demand dynamics *uncertainties* alone effectively reduce lot capacity for visitor traffic by a factor of 3. Finally, we showed how, with appropriate engineering assumptions, one can study tradeoffs among system parameters and results.

We still need more work on evaluating, refining and implementing our models for engineering realistically-sized and, possibly more generally configured, commuter parking lots in the presence of *various* uncertainties. In Pack (2015), we leverage our experience with MU's data and lot operations to begin to look at Engineering Accuracy, while revisiting some issues raised in this paper. Specifically, we calculate and contrast, for MU's **Lot B** ($N=228$), performance statistics, both *directly* from MU measurements and *indirectly* from our new *equilibrium* queueing models, with *estimated parameters*. We emphasize that, because uncertainties usually reduce system capacity or utilization for a given objective, it is important to account for (hedge against) their effects in decision making. New data technologies, e.g., intelligent cars, sensor networks and mobile wireless, will improve lot operations and engineering methods. Thus, to support future research and practice, it may be helpful to develop standards for commuter parking lots that include: service objectives; measurement requirements; queueing models; and engineering techniques.

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