Student ranking by means of non-linear mathematical optimization of participation marks

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Abstract

Ranking students according to their growing participation marks could encourage them to develop self-motivational attitudes toward their academic progress. This paper describes the ranking of students by means of a non-linear mathematical optimization model. The model uses optimization techniques to find the best weights for calculating students' participation marks, effectively eliminating manual evaluation of all possible equations. The optimal weights are used to calculate the average participation marks and the students are ranked accordingly.

Introduction

In recent years, an attitude of indifference among students relating to their academic performance has been noted in Computer Science (CS) education (Du Toit, 2015). Although the common perception among students is that their main responsibility is partaking in lectures, they feel that lecturers have to provide guidance and motivation for them to achieve (Geçer, 2013). The role of the lecturer has changed from just teaching and controlling the learning process to following and facilitating it. A successful lecturer creates an environment in which students harbor a responsibility towards a positive class atmosphere (Ozel, 2015). However, a great obstacle in these classes which inhibits a lecturer's ability to inspire students is the large student numbers which have to be dealt with (Hattie, 2005).

Studies show that the implementation of self-monitoring initiatives in mainstream classes, result in increases in students' participation and subsequent academic achievement (Rock, 2005). Students that self-motivate exhibit a sense of self-management and achieve greater academic success (Di Domenico & Fournier, 2015). Finn and Zimmer (2012) found that a higher level of student engagement in learning, leads to greater academic achievement. Students who try to self-regulate their studies display a sense of responsibility in terms of controlling and performing actions relating to their studies (Garcia, 1996). Barker and Garvin-Doxas (2004) argue that using a hierarchy structure in a class can assist students in understanding improvable factors which determine their academic success and will also enable a lecturer to identify students that are underperforming.

Numerous different methods have been developed in the ranking of individuals. Gao *et al* (2015) followed a fuzzy hierarchical approach to evaluate college sport coaches by using an analytic hierarchy process to determine the appropriate weight levels for each influential factor. After that they employed a fuzzy comprehensive evaluation, derived from the fuzzy set theory (Gottwald, 2010), to calculate a grade or ranking for each coach. Another example is the ranking of the top 150 National Basketball Association players by Mertz (2015). He used linear regression techniques in a statistical model to determine the factors which will have the greatest impact on a player's ranking.

A study by Du Toit (2015) involved the implementation of a computerized mathematical program used to inform students on their academic progress during the course of a semester. The program empirically calculated sets of participation marks for each student, using different combinations of a general mathematical equation and ranked the students according to the average of each set. The students were presented with their class ranking at regular intervals.

Lecturers usually predetermine a mathematical equation to be used for calculation of the participation marks of students. These marks also present some indication of the level at which students participate in their subjects and are eventually used to determine whether a student will be allowed to write exam or not. The purpose of the system developed by Du Toit (2015) was to facilitate the ranking of students by considering all the different options available for the calculation of participation marks. To pass a CS module, a student needs to achieve certain module outcomes in the specific subject. However, because of the nature of CS education, student performance and progress are determined not only through theoretical assessments but also practical assignments, class tests, exams and participation in class activities. Lecturers therefore require a high level of student participation during the majority of the classes to be able to sufficiently prepare them for summative assessments, but struggle to motivate students effectively.

The aim of this research was to develop and solve a mathematical model that:

- Uses non-linear programming techniques to find the best weight distribution, for varying numbers of factors, in calculating participation marks in undergraduate CS modules;
- Calculates the average participation marks of students using the optimized weights; and
- Ranks the students according to their average marks.

The ranking program implemented by Du Toit (2015) is briefly outlined in the next section. This is followed by a discussion on the development and solution of the non-linear mathematical ranking model. The paper ends with a section describing future developments and the incorporation of data envelopment analysis (DEA) in the ranking process, as well as some concluding remarks.

Development of the non-linear mathematical ranking model

The content in CS modules has the feature that new concepts are taught based on the preconception of existing knowledge among the students. So if students neglect to attend class, certain basic concepts are not mastered. Lecturers usually have insufficient insight into student participation from the very start of a new semester. This means that a lecturer cannot always identify students who fall behind early on. More insight into the predicted average- and minimum participation marks of each student can help the lecturer to timeously recognize individuals who are at risk of falling behind in such a way that they will not be able to complete the module successfully. A system that provides such insights to lecturers can also be used to inform the students on their progress from the very start of the semester and present an incentive to perform better.

In general, lecturers require the following insights into student progress, at regular intervals:

- The minimum possible participation mark for each student;
- How each student performs relative to the rest of the class based on student ranking;
- What the expected average participation mark for the entire class is; and
- What the expected throughput of the class is in terms of exam admission.

Students are granted admission to exams if they have a participation mark of forty percent or more (Du Toit, 2015). Factors, that are considered in the calculation of participation marks can include, among others, average scores per student for class attendance, practical assignments, informal tutorial tests, and formal semester tests. Formative tutorial tests, during which students are also encouraged to discuss basic concepts with their peers, are often given in class to assess basic theoretical knowledge needed to complete practical assignments. Summative semester tests are used to determine the level of proficiency with which individual students have mastered theoretical and practical subject content. The impact that each of these factors need to have on the participation mark sometimes vary according to the preference of the specific lecturer, because of different teaching and learning methods employed. Lecturers adhere to certain weight guidelines when calculating the participation marks in CS and would typically inform students how the participation mark for a module is to be calculated. Du Toit's (2015) model can be formulated as follows:

$$P = s_1x_1 + s_2x_2 + \cdots + s_nx_n \quad (1)$$

$$\begin{aligned} \mathit{Subject\ to} \quad & \textstyle\sum_{i=1}^{n} x_i = 1 && (2) \\ & x_i \bmod k = 0 && (3) \\ & l \le x_i \le u && (4) \\ & 0 \le s_i \le 1 && (5) \\ & x_i \ge 0 && (6) \\ & l, u \ge 0 && (7) \end{aligned}$$

where P is the participation mark, s_i is the average score for factor i, x_i is the weight assigned to factor i, (3) ensures that x_i is a multiple of k, l is the lower limit for all the weights, u is the upper limit for all the weights, n is the number of factors used, and $i = 1, \dots, n$.

In undergraduate modules, it is generally accepted that no one single factor should carry a weight of more than forty percent of the resulting participation mark (Du Toit, 2015). Also, weights assigned manually to the factors are generally in multiples of five. In the described scenario, the following values will therefore apply: $k = 0.05, l = 0.05$, and $u = 0.4$.

The ever-changing nature of CS regularly triggers lecturers to implement new types of assessments in their classes (Landau *et al*, 2014). This means that during the course of a semester, a lecturer can decide to add a new type of assessment and want to let it count as another factor in calculating the participation mark. Although the inclusion of such new innovations can benefit the students, the lecturer is then bound by the disclosure made at the beginning of the semester regarding the calculation of the final participation marks. So stipulating a general equation to be used for the participation marks would allow the lecturer to determine the specific factors during the course of the semester and also enable the students to control the weights by improving in the areas still available to them.

Du Toit's (2015) ranking program uses the model in (1)-(7) to calculate a participation mark for every combination of different weight levels that are allowed by the constraints. This means that for each factor used in a calculation, there are 8 ($= \frac{0.4}{0.05}$) or less weight levels that may apply and the sum of all the weights must add up to 1 (100%). So, for each factor a lecturer decides to include in the calculation of the participation marks, the number of equations available due to adding and redistributing weights, increase exponentially. The total number of possible equations available for the number of factors from 3-12 was determined empirically and is shown in Table 1.

Note that because the use of one or two factors in the calculation of participation marks would violate constraint (2), these scenarios are not considered. Apart from the fact that calculating 315 different sets of marks per student for only four factors manually is impractical, the scenarios available in these equations do not consider that some factors could be more important than others. For instance, generally a semester test average must bear more weight than the class test average, and the class test average is more important than attendance.

The following additional constraint is therefore added to the model:

$$x_i \geq x_{i+1}, \quad i < n \tag{8}$$

The addition of the constraint in (8) saw the number of possible equations for participation mark calculation decrease considerably (see Table 2). For three factors, and with the inclusion of constraints (2)-(8), Du Toit's program (2015) would typically calculate a total of four participation marks per student, using the equations given in Table 3.

The program would then determine the minimum-, maximum-, and average participation marks, and rank the students according to the average marks. An example set of calculated marks for six students, is displayed in Table 4.

Table 1. Number of available equations for participation marks calculation

Factors (n)	*Number of equations*
3	15
4	315
5	2 226
6	8 856
7	23 898
8	47 748
9	74 097
10	91 828
11	92 257
12	75 570
Total	416 810

Table 2. Number of available equations for participation marks calculation with the additional constraint

Factors (n)	*Number of equations*
3	4
4	23
5	46
6	61
7	63
8	58
9	47
10	38
11	28
12	21
Total	389

Table 3. Available participation mark equations for $n = 3$ factors

Equation 1	$0.4s_1 + 0.3s_2 + 0.3s_3$
Equation 2	$0.35s_1 + 0.35s_2 + 0.3s_3$
Equation 3	$0.4s_1 + 0.35s_2 + 0.25s_3$
Equation 4	$0.4s_1 + 0.4s_2 + 0.2s_3$

Table 4: Participation marks per student using all available equations

	Equation 1	*Equation 2*	*Equation 3*	*Equation 4*	*Minimum*	*Average*	*Ranking*
Student A	60%	59%	61%	63%	59%	60.84%	4
Student B	91%	92%	91%	91%	91%	91.40%	1
Student C	81%	80%	81%	81%	80%	80.89%	2
Student D	53%	54%	54%	55%	53%	53.99%	6
Student E	64%	65%	65%	66%	64%	65.05%	3
Student F	58%	59%	58%	57%	57%	58.31%	5

Additional statistics provided by the program include the throughput figures and average class participation mark for each available equation. These numbers assisted the lecturer in deciding what the best weight distribution for the factors would be and in providing minimum participation marks and rankings to students. Students had insight into their progress in the module but were also able to systematically increase the average participation mark *en masse* by working harder in those factors on which they could still have an impact.

The program also presented the lecturer with additional class statistics, as shown in Table 5.

Table 5: Class statistics for the four different equations

Equation	*Number of students with no exam admission*	*Average class participation mark*
1	12	54.08%
2	12	55.54%
3	13	54.82%
4	17	52.62%

In this example, the statistics enabled the lecturer to see the effect of using the various equations on the resulting number of students without exam admission, and on the average participation mark. Feedback received from the students in the class was overwhelmingly positive. Most of them felt that knowing their exact rank in relation to the rest of the class, motivated them to work harder in the areas which they knew could still cause their positions to improve. Although successful, the system effectively searched for the best weights to use by empirically calculating all of the possibilities and manually selecting one option. The purpose of the proposed model was to find the optimal weights and calculate the participation mark only once. To develop a model that optimizes the calculation process, certain system requirements needed to be considered. These requirements include to:

- Determine the minimum- and maximum participation mark for each student;
- Determine the average weight for each factor and use them to calculate the average participation marks;
- Rank the students based on their average participation marks; and
- Determine the resulting throughput.

The mathematical model implemented to optimize this problem can therefore be written as follows:

$$Maximize/minimize \quad P = \sum_{i=1}^{n} s_i x_i \tag{9}$$
$$Subject\ to \quad \sum_{i=1}^{n} x_i = 1 \tag{10}$$
$$x_i \bmod k = 0 \tag{11}$$
$$l \leq x_i \leq u \tag{12}$$
$$x_i \geq x_{i+1}, \qquad i < n \tag{13}$$
$$0 \leq s_i \leq 1 \tag{14}$$
$$x_i \geq 0 \tag{15}$$
$$l, u \geq 0 \tag{16}$$

where P is the participation mark, s_i is the average score for factor i, x_i is the weight assigned to factor i, (11) ensures that x_i is a multiple of k, l is the lower limit for all the weights, u is the upper limit for all the weights, n is the number of factors used, and $i = 1, \ldots, n$.

Implementation of the mathematical ranking model

The model was implemented in an Excel spreadsheet and, due to the non-linearity caused by constraint (11), was solved utilizing the Evolutionary Solver (Baker, 2011). This procedure generally finds heuristic solutions when optimal answers are not guaranteed. The score data set implemented in Du Toit's (2015) program (Table 6), was used to obtain the maximum-, minimum- and average participation marks per student. The data set contained the average scores of three factors, for six students. The factors represented a semester test (s_1), class test average (s_2), and average score for practical assignments (s_3). For each of the students in Table 6, the participation mark was maximized and minimized using Evolutionary Solver. These results are shown in Table 7.

Table 6. Mark set used in the non-linear optimization program

	s_1	s_2	s_3
Student A	66%	73%	37%
Student B	97%	84%	93%
Student C	78%	87%	77%
Student D	70%	49%	37%
Student E	71%	66%	55%
Student F	64%	47%	65%

Table 7. Results achieved by non-linear optimization program

Student	*Max. participation (%)*	*Min. participation (%)*
A	63.0000	59.4000
B	91.9000	91.2500
C	81.4000	80.4000
D	55.0000	52.7500
E	65.8000	64.4500
F	59.2000	57.4000

The results obtained yielded the same maximum- and minimum participation marks when compared to Du Toit (2015) in all instances but one, where the difference was negligible. The model provided the average weights (x_i) for each of the factors (s_i), which were used to calculate the average participation mark per student and to determine the student ranking. The resulting ranking was the same as those achieved by Du Toit (2015).

Future developments and conclusions

Optimizing the system further by implementing DEA principles is currently being investigated. DEA is a method used to measure the efficiency of decision making units (DMUs) with multiple inputs and –outputs, relative to one another (Seiford & Zhu, 2003). It is commonly implemented in cases where different operating units need to be compared with one another and can be solved as sets of linear programs (one for each DMU), also described in Adler *et al* (2002). Determining fair and effective inputs where students are concerned can be very difficult. A method will be investigated, similar to the one used by Kao and Lin (2008) who used DEA without inputs to allocate universities to classes of differing levels. All universities in the same class are considered equally efficient and better than those in lower classes. Their method determined the Pareto optimal universities by implementing the following model:

$$Maximize \quad E_k = \sum_{j=1}^{m} Y_{kj} w_j \tag{17}$$

$$Subject\ to \quad \sum_{j=1}^{m} Y_{ij} w_j \leq 1 \qquad i = 1, \ldots, n \tag{18}$$

$$w_j \geq \varepsilon > 0, \qquad j = 1, \ldots, m \tag{19}$$

where E_k is the composite index, Y_{ij} is the measure of unit i in criterion j, w_j is the weight for criterion j, n is the number of units to be evaluated, m is the number of criteria, and ε is a small positive number.

This model provides only the ultimate goal, so a DMU is ranked high if it performs very high in one criterion. When the DMUs are students, the nature of ranking and the relationship between the criteria, prompts the use of the dual formulation of this model, given in (20)-(22), which will present each student with intermediate targets to reach for improving his/her position in the ranking.

$$Maximize \quad E_k = \sum_{i=1}^{n} \lambda_i - \varepsilon \sum_{j=1}^{m} s_j \tag{20}$$

$$Subject\ to \quad \sum_{i=1}^{n} Y_{ij}\lambda_i - s_j = Y_{kj} \qquad j = 1, \dots, m \tag{21}$$

$$\lambda_{ji}, s_j \geq 0, \qquad i = 1, \dots, n;\ j = 1, \dots, m \tag{22}$$

where E_k is the composite index, Y_{ij} is the measure of unit i in criterion j, w_j is the weight for criterion j, n is the number of units to be evaluated, m is the number of criteria, ε is a small positive number, $\theta = \sum_{i=1}^{n} \lambda_i$, $(Y_{kj} + s_j)/\theta$ is the target, and $j = 1, \dots, m$.

In conclusion, this paper discussed the development of a non-linear mathematical model which was successfully solved to determine the best weight distribution for existing score data sets. The model was implemented in CS modules, enabling the lecturer to provide the students with regular performance updates by informing them on their progress and ranking in relation to their peers. The reaction of the students was very positive. It gave them a sense of control over their own progress and compelled them to improve their positions, creating a self-motivational class setting.

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