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Regularity patterns for rolling stock rotation optimization

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Abstract

The operation of railways gives rise to many fundamental optimization problems. One of these problems is to cover a given set of timetabled trips by a set of rolling stock rotations. This is well known as the Rolling Stock Rotation Problem (RSRP). Most approaches in the literature focus primarily on modeling and minimizing the operational costs. However, an essential aspect for the industrial application is mostly neglected. As the RSRP follows timetabling and line planning, where periodicity is a highly desired property, it is also desired to carry over periodic structures to rolling stock rotations and following operations. We call this complex requirement regularity. Regularity turns out to be of essential interest, especially in the industrial scenarios that we tackle in cooperation with DB Fernverkehr AG. Moreover, regularity in the context of the RSRP has not been investigated thoroughly in the literature so far. We introduce three regularity patterns to tackle this requirement, namely regular trips, regular turns, and regular handouts. We present a two-stage approach in order to optimize all three regularity patterns. At first, we integrate regularity patterns into an integer programming approach for the minimization of the operational cost of rolling stock rotations. Afterwards regular handouts are computed. These handouts present the rotations of the first stage in the most regular way. Our computational results (i.e., rolling stock rotations evaluated by planners of DB Fernverkehr AG) show that the three regularity patterns and our concept are a valuable and, moreover, an essential contribution to rolling stock rotation optimization.

Introduction

Railway vehicles, known as rolling stock, are expensive as well as limited assets of a railway operator. Given a timetable, the rolling stock fleet must cover the trips of the timetable in the most efficient way in order to maximize the revenue of the railway operator.

Rolling stock rotations are used as a master plan for a medium term period in the railway planning process. Thus, they are developed in advance. In our applications at DB Fernverkehr AG trips of the timetable repeat from week to week. Hence, we consider a cyclic timetable for one week, which is called *standard week*. The rolling stock rotations are cycles that cover timetabled trips for the purpose of deciding what activity a dedicated rail vehicle performs after the operation of a timetabled trip. These decisions are crucial for the efficiency and must absolutely comply with several intricate conditions:

- vehicle composition rules,
- turn time rules,
- routing and deadhead rules,
- maintenance constraints, and
- infrastructure capacities.

This variety of requirements gives rise to a very challenging competition on rolling stock rotation planning. Our *productive optimization software* RotOR participates in this competition for one of the leading railway operators in Europe: DB Fernverkehr AG, see Borndörfer *et al.(*2015).

Obviously, the main goal in rolling stock rotation planning is to minimize the operational cost. The most important properties of the operational cost are the number of vehicles used to cover a given timetable and the cost for deadhead trips that must be allocated to change the location of rolling stock vehicles whenever it is necessary or efficient.

DB Fernverkehr AG aims to provide a periodic timetable to the passengers. Consequently, trips of a single train mostly only differ w.r.t. the day of operation in the provided scenarios. This means that the departure and arrival locations and the corresponding times are equal. Operating trains that do not differ over the operational days is a desired result in timetabling (Liebchen & Möhring (2007)) and, therefore, also yields a quality measure for a timetable. The recurring trips are the connection of timetabling and rolling stock rotation planning. Thus, regularity is also desired for the latter. The reasons for the necessity of regularity are manifold. A major reason is the direct impact on the passengers, who prefer a regular operation of trains (Figure 1 shows the opposite). Furthermore, rolling stock rotations are the basis for further planning steps, e.g., crew scheduling and dispatching. In all of these planning steps it is desired to work with simple structures, which are easy to communicate and operate for the daily application. Prominent examples are regularity in airline crew scheduling or freight railway crew scheduling, see Klabjan *et al.*(2001) and Juette *et al.* (2011).

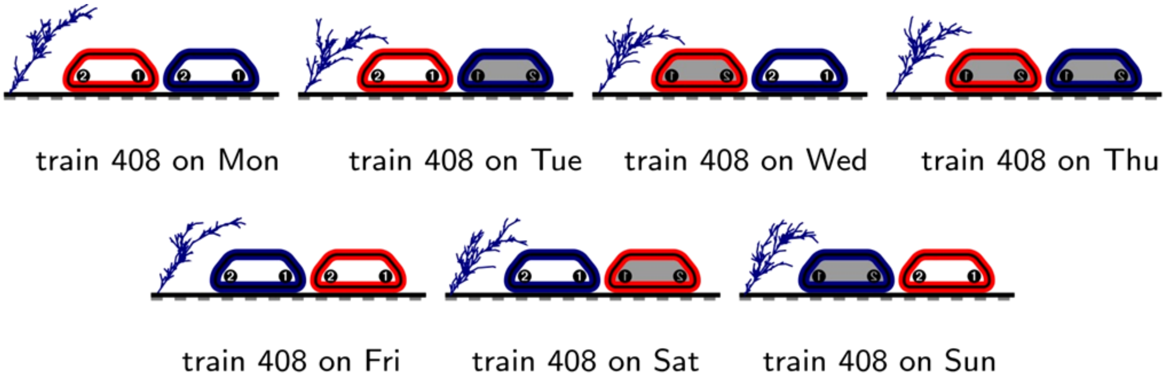

Figure 1. Most irregular operation of train 408

In the next section we introduce the mathematical concept of regularity patterns, which are a powerful modeling tool in order to provide control of regularity within the rolling stock rotations. The Section headlined “The Two Stage Approach” describes the algorithmic concept, i.e., a two stage approach optimization connected via regular hyperarcs, to integrate regularity patterns in the rolling stock rotation optimization framework RotOR. Finally, we report on the benefit of the provided results in practice at DB Fernverkehr AG in the last section.

The concept of regularity

A very simple example of regularity patterns is provided in Figure 1: We consider a train with train number 408. We assume that train 408 is operated by seven different vehicle compositions, i.e., the vehicle composition is different on each day of operation and the pattern is, in a way, as irregular as possible. In contrast, it is desired to operate train 408 with the same vehicle composition on each day of operation in rolling stock rotation planning.

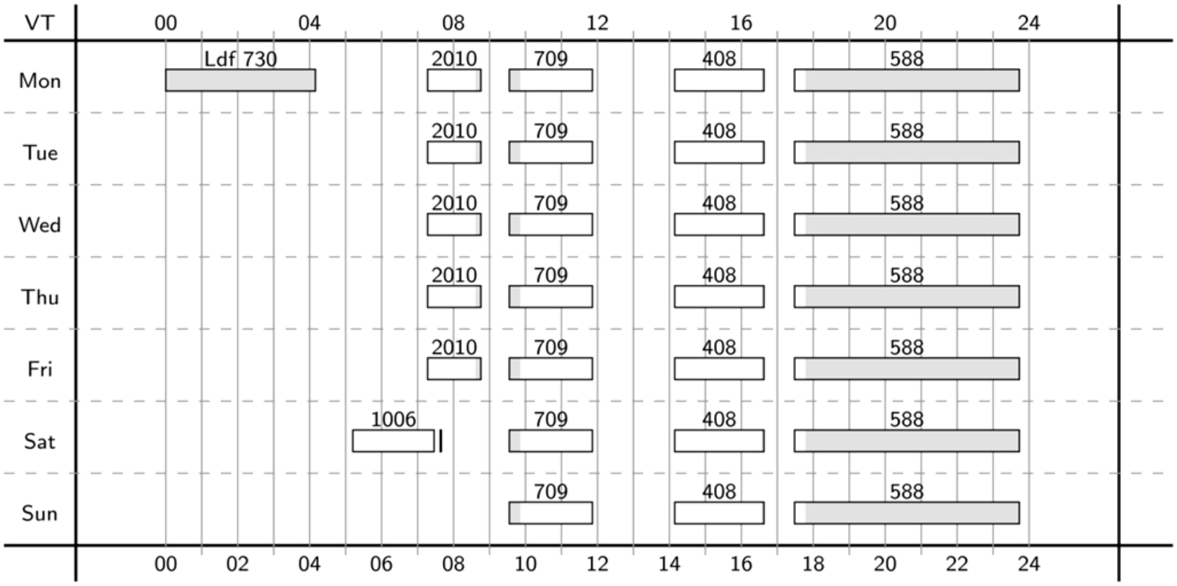

Figure 2. Seven segments of a rolling stock rotation visualized as a block of a handout

Figure 2 shows part of an exemplary rolling stock rotation plan. Each of the seven paths determines how timetabled trips succeed, i.e., *turn*, in the rolling stock rotations for one day of operation. The figure illustrates all of the three regularity patterns that we want to appear as much as possible:

Regular trips: Timetabled trips that have the same train number (the numbers denoted above the boxes) are operated with the same vehicle orientation – gray means: The vehicle is operated with the first class in front (called *Tick*) and white means that the second class is in front of the vehicle (called *Tack*).

Regular turns: The train numbers of successive timetabled trips follow patterns, e.g., the trips of train 408 always turn into trips of train 588.

Regular handouts: The arrangement of the seven paths shows many similarities between the individual paths, e.g., all trips of trains 408, 588, and 709 appear in the arrangement in one block.

The two-stage approach

The main contribution of the paper is a two stage approach that is implemented in RotOR. This allows us to integrate very sophisticated patterns into the rolling stock rotations. The idea is to extend the hypergraph based algorithm by a post processing stage. In this second stage, we solve a quadratic assignment problem (Pardalos *et al.* (1994)) by a fast combinatorial heuristic that is based on the Hungarian method (Kuhn (1955)) followed by a Kernighan-Lin improvement heuristic (Kernighan & Lin (1970)). This procedure leads to rolling stock rotations with a sufficient amount of regularity patterns and to handouts visualizing the rotations such that they can easily be implemented in practice.

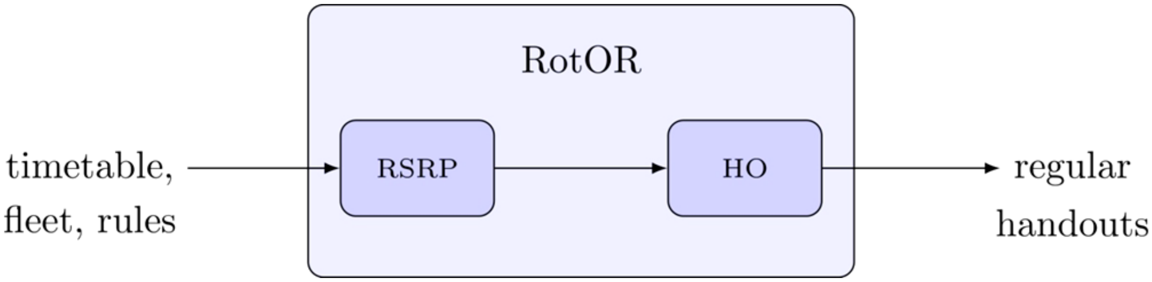

Figure 3. Two stage solution approach of RotOR

We tackle the RSRP by a mixed integer program that is based on a hypergraph model. In a nutshell, to solve these instances the LP-relaxation is solved by a branch and price procedure. Subsequently, a sophisticated branch and bound scheme is used to construct integer solutions. We refer the reader to Borndörfer *et al.* (2015) for a detailed description of the hypergraph model and the algorithm.

In the hypergraph vehicle operations are modeled as *vehicle arcs* that connect the arrival of a train *a* to the departure of a train *b* in case of a turn and connect the departure of train *a* to the arrival of train *a* in case of a trip. The operation of a trip (as well as a turn) by more than one physical vehicle is modeled by a hyperarc that consists of the vehicle arcs of the required individual vehicle movements. Regular turns and trips are modeled by *regular hyperarcs*. These hyperarcs consist of vehicle arcs which correspond to the same turn or trip but to different days of operations. For the example shown in Figure 2 the hypergraph contains (among others) seven vehicle arcs connecting the arrival of 408 to the departure of 588 and a regular hyperarc that consists of all these seven vehicle arcs. In order to reward regular hyperarcs in the resulting rolling stock rotations all non-regular hyperarcs are penalized by a predefined cost parameter times the number of vehicle arcs contained in the corresponding hyperarc. In this way the first two regularity patterns of the previous section are directly integrated in the solution process of the RSRP.

The remaining regularity pattern, i.e., the regular handouts, is handled in the second stage of the solution process. At the point in time where optimized rotations are at hand they have to be transferred from a cycle in a hypergraph to a format used in the railway industry, the *handout*. To transfer a rotation to a handout the rotation is split into segments that contain a consecutive sequence of turns and trips on a fixed day of operation. These segments are grouped into numbered blocks such that each block contains a segment for each day of the standard week. There are two objectives that should hold for this ordering process. First, two segments in the same block should be similar and, second, segments of a fixed block should be succeeded in the next block. This defines an optimization problem which is called Handout Optimization Problem (HO) and can be modeled as a quadratic assignment problem(Pardalos *et al*. (1994)). We solve this problem by a very fast combinatorial heuristic based on the solution of iterated assignment problems through the Hungarian method (Kuhn (1955)) to construct feasible solutions followed by an improvement heuristic based on a Kernighan-Lin (Kernighan & Lin (1970)) heuristic. We remark, that there are different handouts for a fixed rotation with of course different objective values with respect to the objective function of model *HO*.

Additionally, the objective of model *HO* - which is to minimize the sum of all differences between segments in the same blocks and the number of segments that are not succeeded in the next block - is used as a quality measure of different rotations as they directly show how many differences between segments of the the same block occur. As mentioned, having as few as possible differences is a highly desired property in rolling stock rotation planning.

Results in practice

Table 1 shows the characteristics of rolling stock rotation solutions of a typical real world instance from DB Fernverkehr. This instance covers 788 trips and its hypergraph contains roughly 10 million hyperarcs. The characteristic of four solutions gives an example of a very typical effect of the introduction of regularity patterns to the RSRP. The first column of the table marks if the first two regularity patterns of Section "The Concept of Regularity" were considered in the hypergraph, i.e., regular hyperarcs were present, or not. For a fair comparison, the second column contains operational costs without costs for regularity. In the last two columns the objective function values of *HO* for the solutions given by the heuristic and for the optimal solution are given. In the latter case model *HO* was solved to optimality with CPLEX 12.6 1with the heuristic solution as warmstart.

Table 1. Effects of regularity on the solution process of a typical instance.

Regularity Patterns	*Operational Cost*	*Heuristic Handout Cost*	*Exact Handout Cost*
No	2260069	3941	3782
Yes	2271942	2269	2032

The operational costs of the solutions are of course higher in the case where regularity is included. But, it is a very slight increase of roughly 0.5%. In return, the regularity measurement given by the objective function of *HO* is significantly improved by more than 70%. It is also a general behavior that the heuristic provides high quality solutions in the single-digit percentage area.

Summarized, the framework RotOR is able to produce regular handouts as visualized in Figure 2. All crucial information to operate the rolling stock rotations are provided for the railway undertaking. Thus, the planners at DB Fernverkehr AG can immediately evaluate if the regularity of a rolling stock roster plan is adequate or if it should improve at the expenses of other aspects, which are usually additional cost for turn around or deadhead trips or in the worst case cost for additional maintenance operations or even vehicles.

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